CHORD THEOREMS IN THE GEOMETRY OF A CIRCLE

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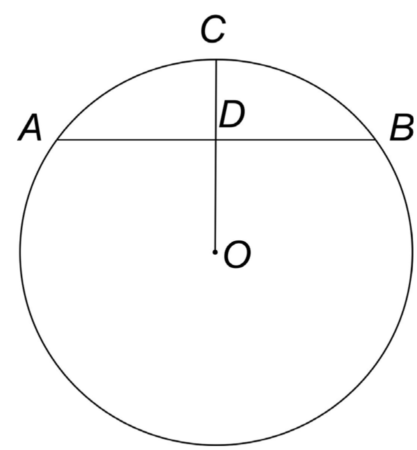
Recall that a circle is the locus of points in a plane that are at a constant different from a given point in a plane and that the radius of a circle is a line segment[1] that connects the center of a circle with any point on its circumference.

Since a circle can have an infinite number of radii, all radii of a circle are equal in length.



A **chord** of a circle is a line segment that joins any two distinct points that lie on the circumference of a circle. If one of these line segments passes through the center of a circle, then that chord has a special name; it is the **diameter** of a circle.

Here are two theorems about chords and circles. First, note the figure. If a line, or a line segment, containing the center of a circle is perpendicular to a chord, it bisects it. We let this line be the radius of circle *O*. Therefore, if the radius $\overline{OC}$ of a circle is perpendicular to the chord $\overline{AB}$ (We assert that the chord is not the circle's diameter.), it bisects it; i.e.,

$$\overline{OC} \perp \overline{AB} \Rightarrow AD = DB$$

Conversely, if a line, or a line segment, containing the center of a circle bisects a chord that is not a diameter, it is perpendicular to it. Again, we let this line be the radius of circle *O*. Therefore, if the radius $\overline{OC}$ bisects the chord $\overline{AB}$ (We assert that the chord is not the circle's diameter.), it is perpendicular to it; i.e.,

$$AD = DB \Rightarrow \overline{OC} \perp \overline{AB}$$

CHORD THEOREM 1

If a line, or a line segment, containing the center of a circle is perpendicular to a chord, it bisects it. Or, given our figure, we must prove $\overline{OC} \perp \overline{AB} \Rightarrow AD = DB$ where $\overline{AB}$ is not the diameter of circle *O*.

Given: $\overline{OC} \perp \overline{AB}$

Prove: $AD = DB$

Logic:

Statement	*Justification*
$\overline{OC} \perp \overline{AB}$	Given
$m\angle ADC = m\angle BDC = 90°$	Perpendicular lines generate right angles at the point of intersection.
Draw $\overline{OA}$ and $\overline{OB}$	Two points determine a line.

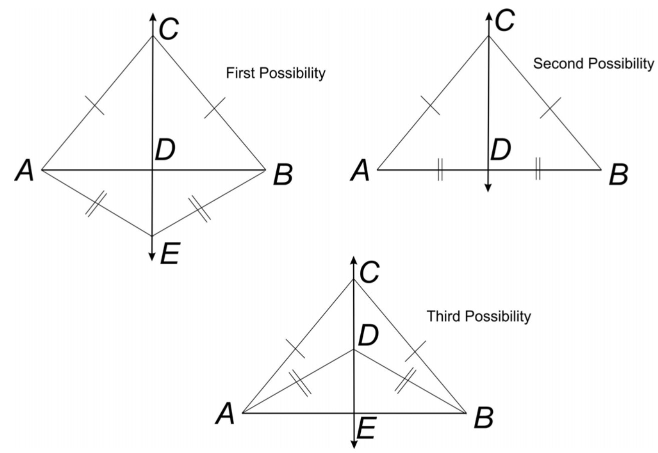
[1] The radius of a circle could refer (1) to its measurable length, a quantity, or (2) the locus of points that make up its length; i.e., a line segment.

Statement	*Justification*
ΔADO and ΔBDO are right triangles.	A right triangle has one interior angle that is a right angle.
$OA = OB$	All radii of a circle are equal.
$OD = OD$	Reflexive property of equality.
$\Delta ADO \cong \Delta BDO$	HL congruence theorem for right triangles. (See Step 13, Lesson 5 homework.)
$AD = BD$	Corresponding parts of congruent triangles are equal.

QED

PREPARATORY THEOREM

In a plane, two points equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. We note that there are three possibilities that we must prove. Inspect the figure.



The following proof establishes the truth of the first possibility.

Given: $CA = CB$; $EA = EB$

Prove: $\overleftrightarrow{CE}$ is a perpendicular bisector of $\overline{AB}$

Statement	*Justification*
$CA = CB$; $EA = EB$	Given
$CE = CE$	Reflexive property of equality.
$\Delta ACE \cong \Delta BCE$	SSS Congruence Theorem.
$m\angle ACE = m\angle BCE$	Corresponding parts of congruent triangles are equal.
$CD = CD$	Reflexive property of equality.
$\Delta ACD \cong \Delta BCD$	SAS Congruence Theorem.
$m\angle CDA = m\angle CDB$	Corresponding parts of congruent triangles are equal.
$m\angle CDA = m\angle CDB = 90°$	We know that $m\angle CDA + m\angle CDB = 180° \Leftrightarrow m\angle CDA = m\angle CDB = 90°$ (since $m\angle CDA = m\angle CDB$ and some simple algebraic operations). In general, if two angles in a linear pair are equal, then they are right angles.
$\overleftrightarrow{CE} \perp \overline{AB}$	If two lines (or line segments, or line segment and a line) form right angles, they are perpendicular.
$AD = DB$	Corresponding parts of congruent triangles are equal.

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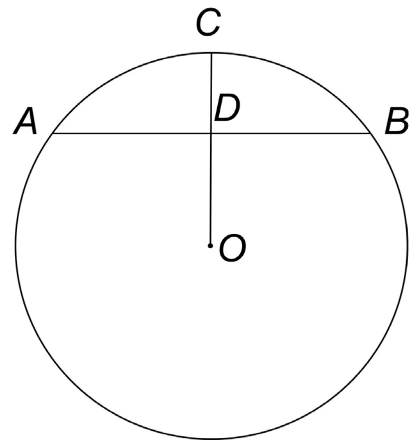
Statement	*Justification*
$\overleftrightarrow{CE}$ bisects of $\overline{AB}$	If a line segment is divided into two equal segments, it is bisected.

Therefore, $\overleftrightarrow{CE}$ is a perpendicular bisector of $\overline{AB}$

QED

CHORD THEOREM 2



If a line, or a line segment, containing the center of a circle bisects a chord that is not a diameter, it is perpendicular to it. Or, given our figure, prove $AD = DB \Rightarrow \overline{OC} \perp \overline{AB}$ where $\overline{AB}$ is not the diameter of circle *O.*

Given: $AD = DB$

Prove: $\overline{OC} \perp \overline{AB}$

Logic:

Statement	*Justification*
$AD = DB$	Given
Draw $\overline{OA}$ and $\overline{OB}$	Two points determine a line.
ΔADO and ΔBDO are right triangles.	A right triangle has one interior angle that is a right angle.
$OA = OB$	All radii of a circle are equal.
$\overline{OC} \perp \overline{AB}$	In a plane, two points equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment.

QED