Restoring the Third "R"

by James D. Nickel

Homework

This essay is an expansion of a short talk given by James Nickel at the *Education Policy Conference 20*, held in St. Louis, Missouri, from 29-31 January 2009.

On a separate sheet of paper, do the following:

Step 1. Choose any three-digit number (each digit must be different).
Step 2. Reverse the digits of the number you selected.
Step 3. Subtract the smaller number from the larger. Make sure that you have a three-digit answer; if two digits, write 0 in the hundreds position and follow the next instruction.
Step 4. Reverse the digits in the difference.
Step 5. Add the last two numbers.

Your answer, if you did your arithmetic correctly, should be 1089. Here is an example of the process.

Source: iStockPhoto

Step 1. 189
Step 2. 981
Step 3.

$$\begin{array}{r} 981 \\ \underline{-189} \\ \not{8}\not{0}2 \\ 792 \end{array}$$

Step 4. 297
Step 5.

$$\begin{array}{r} 792 \\ \underline{+297} \\ 989 \\ \underline{1} \\ 1089 \end{array}$$

989 . See note below.[1]

Why does this always work? I'll leave it to the reader to figure it out. To do this, you must possess a mastery of position notation along with knowledge of the rudiments of algebra.

The purpose of this essay is to explore the issues involving the understanding and doing of arithmetic. Specifically, this essay will:

- Show the importance of arithmetic in the broader scope of mathematics instruction and culture.
- Define the precepts, the first principles, of arithmetic.
- Explore some recent trends in arithmetic instruction that are detrimental to it.
- Define what is meant by "fuzzy" math.
- Give direction for reform in arithmetic education.

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[1] Steps 3 and 5 incorporate a left-to-right paradigm of subtraction and addition. See James Nickel, *The Dance of Number* (forthcoming) and Edward Stoddard, *Speed Mathematics Simplified* (New York: Dover Publications, [1962, 1965] 1994) for detailed explanation and rationale of this methodology.

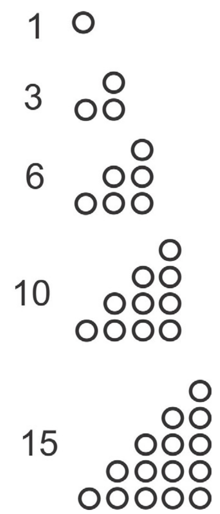
These goals are a tall order, and I can only summarize the main points. We begin at the beginning … definitions.

Preliminary Definitions

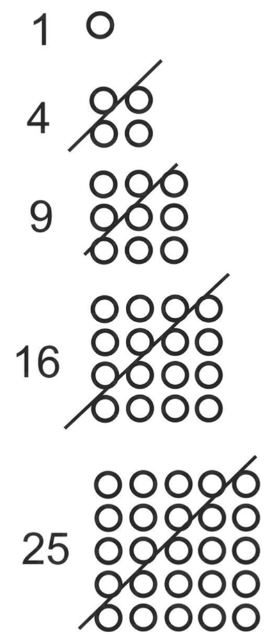
Before we show the importance of arithmetic, let's explore two definitions. It may surprise the reader to discover that, for the classical Greeks, arithmetic did not mean to them what it now means to us.

The Greek word transliterated *arithmetic* means "art or skill." To the ancient Greeks, this skill unfolded in a "technique" that modern mathematicians now called number theory. Ancient Greek philosophers like Pythagoras (582-500 BC) and Plato (427-347 BC) studied and promoted this "techne." Greek mathematicians and philosophers, when studying the nature of number, looked for patterns. Based upon these discovered patterns, they proved, by the method of deduction, many propositions, or theorems, about the way numbers interact with each other.

Let's look at one example. Many number patterns are discernible by arranging stones or pebbles in the sand. The figure at right is a pebble demonstration of triangular numbers. By rearranging these objects, you can observe that a square number is the sum of two successful triangular numbers. The ancient Greeks discovered many types of numbers, the most famous being prime numbers, numbers divisible only by themselves and 1. Prime numbers are the basic building blocks of every non-prime number or composite number.[2] Modern mathematicians have coordinated the theorems about prime numbers to secure the sending and receiving of confidential Internet transmissions.[3]



Triangular Numbers



Square Numbers

The Greek word transliterated *logistica* means to reckon. It incorporated the practice of what we commonly call commercial arithmetic. Ancient Greek philosophers did not bother with *logistica*; it was the province of Greek slaves. These slaves used a counting board to do these calculations. This board was the precursor of the abacus and, eventually, the modern calculator.

The twin streams of theory and calculation have been meandering through the landscape of mathematics ever since. The teaching of the rudiments of arithmetic must drink from both watercourses, with a necessary emphasis on calculation supported by number theory, especially in the teaching of the arithmetic of fractions.[4]


Counting Board (Public Domain)

The Importance of Arithmetic

Before we summarize the components that incorporate the rudiments of arithmetic, I will explain the importance of arithmetic in the broader scope of mathematics instruction.

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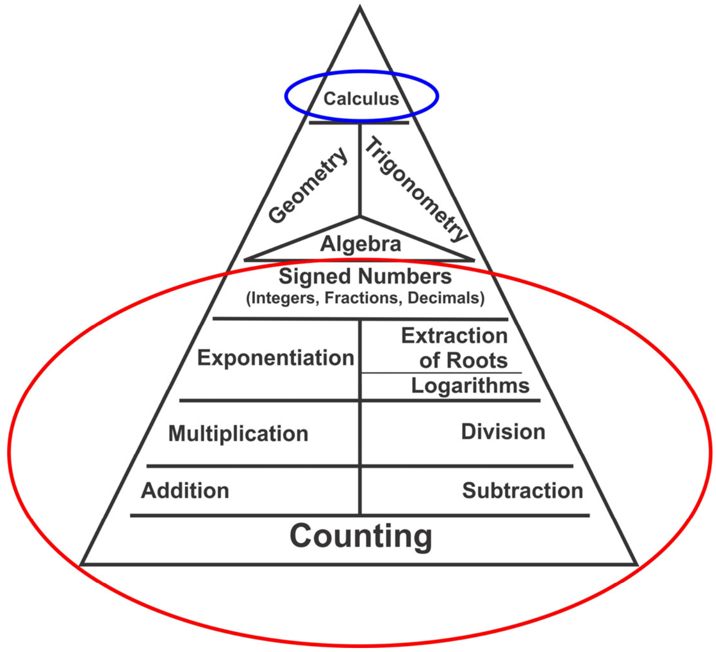
[2] The *Fundamental Theorem of Arithmetic*, basic to modern number theory, states that every composite number can be equated to a *unique* product of prime numbers. For example, $36 = 2 \times 2 \times 3 \times 3$ (2 and 3 are prime numbers).

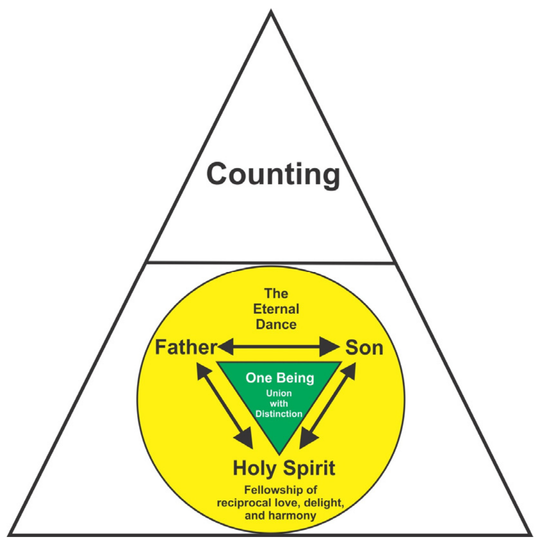
[3] I am speaking of the RSA security algorithm.

[4] For example, in the arithmetic of fractions, the student must invoke number theory to find the least common denominator of two or more fractions.

In the figure at right, I have circled the "rudiments" in red. Note the bottom component: counting. The operations of arithmetic unfold from the simple process of counting. For example, given the counting numbers {1, 2, 3, 4, …}, addition is counting forward, multiplication is counting forward faster, subtraction is counting backward, and division is counting backward faster.



Since counting is the foundation of arithmetic, we should also ask the question, "What is the foundation of counting?" The answer to this question is a trail into the forest of philosophy, a land into which mathematicians, at times, have ventured but without much success. The philosopher and mathematician Alfred North Whitehead (1861-1947) once said, "It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profundity, he is talking nonsense."[5]



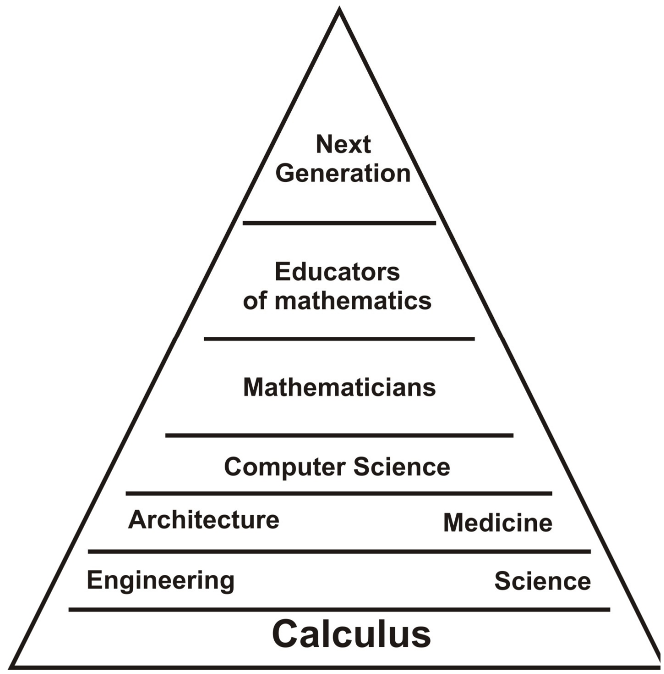
For the Biblical Christian, the answer to this question lies in the onto-relational God revealed by Jesus Christ. It is this God who created man in His image, and that image includes the ability to think, count, and measure. God thinks, but His thoughts are above man's thoughts (Isaiah 55:8-9). God counts and names the stars along with the number of hairs on our head (Psalm 147:4; Isaiah 40:26, and Matthew 10:30), but this counting is not sequential. God does not count "1, 2, 3, etc."); He knows number and name. God marks off the heavens with a span (Isaiah 40:12-13), but He does not need a yardstick to do this. Augustine (354-430) wrote that man, made in God's image, is capable of "thinking God's thoughts after Him." God's word, who is Jesus (John 1:1-14), as revelation of His

[5] Alfred North Whitehead, *An Introduction to Mathematics* (New York, 1911), p. 227.

thoughts, is the proper starting point for man's thinking. Likewise, man counts and measures after God. The question of why man can count is, ultimately, a philosophical question, which is, in the final analysis, a theological question. We can only account for counting by looking to the Triune God revealed in Scripture.

Note the apex of the triangle: calculus (circled in blue). We start at the base, the ground: the Triune God is the foundation of counting. Next, we see that counting is the foundation of arithmetic, arithmetic is the foundation of algebra, and algebra, along with geometry and trigonometry, is the foundation of the Calculus.[6] The historian Arnold Toynbee (1889-1975) notes the significance of the Calculus:

> Looking back, I feel sure that I ought not to have been offered the choice [whether to study Greek or calculus–JN] … calculus ought to have been compulsory for me. One ought, after all, to be initiated into the life of the world in which one is going to live. I was going to live in the Western World … and the calculus, like the full-rigged sailing ship, is … one of the characteristic expressions of the modern Western genius.[7]



The figure at right illustrates how the Calculus is the rigging of the ship called modern technology. It is also the foundation of many higher branches of mathematical study. The figure also shows that the Calculus forms the knowledge base of mathematics educators, men and women entrusted with teaching the next generation of students of mathematics.

The reader should now recognize the importance of the mastery of arithmetic. Without it, you cannot master the algebra. Without algebra, you cannot master of the Calculus. Without the Calculus, you cannot maintain or advance modern technology. Also at stake is the transmission of the fundamentals of arithmetic to the next generation.

The Precepts of Arithmetic

Several conceptual ideas ground the precepts of arithmetic. The first, and most important, is our decimal positional system, a legacy from India. The French mathematician Pierre-Simon de Laplace (1749-1827) notes its significance:

> "Blessed are the placemakers, for they shall be called the children of God."
> Matthew 5:9 (misprint from a Bible printed in 1562).

> It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to computations put our arithmetic in the first

[6] Calculus is the quantitative study of instantaneous rates of change. The word Calculus is Latin meaning "pebble or stone." These objects were used in ancient times to count!

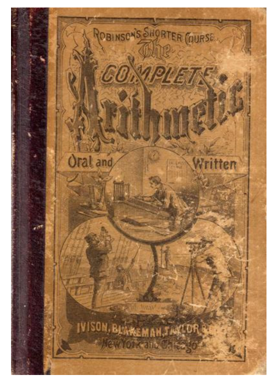
[7] Arnold Toynbee, *Experiences* (New York: Oxford University Press, 1969), pp. 12-13.

rank of useful inventions; and we shall appreciate the grandeur of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.[8]

Ease of computation is a derivative of this base ten positional system of notation.[9] As we have already noted, the four foundational operations are extensions of counting forward and backward. The logician and mathematician Charles Dodgson (1832-1898), aka Lewis Carroll, once identified these operations as:

- Ambition,
- Distraction,
- Uglification, and
- Derision.

Counting objects requires the counting numbers. To measure requires more. Ask any carpenter if he can do his work using counting numbers alone. Thus, measurement requires the mastery of the arithmetic of fraction and decimals. Fractions and decimals naturally unfold the meaning and use of ratios, proportions, and percents.



Regarding the teaching of the precepts of arithmetic, you will find it an enlightening exercise to compare today's textbooks with books used a century ago. Try to procure *Robinson's Shorter Course, The Complete Arithmetic: Oral and Written* (New York and Chicago: Ivison, Blakeman, Taylor & Co., 1874), written by Daniel W. Fish, A.M. Then, get *Everyday Mathematics* (The University of Chicago School Mathematics Project[10]) and start comparing.

I will suspend my judgment on the pedagogy of modern arithmetic instruction until later. Fish's textbook is dated by the times in which it was written. Today's textbooks are also dated by the times. By that, I mean that political correctness, feminism, and multiculturalism are the colors that paint many of the problems and illustrations in arithmetic textbooks published by the major textbook publishing companies.

Instead of indoctrinating political correctness, arithmetic texts should be teaching their students the thinking skills that go with the computational skills. I have extracted the following observations from Thomas Sowell, renowned economist, to illustrate this thesis.[11]

- *A* can always exceed *B* if not all of *B* is counted and/or if *A* is exaggerated.
- Most variables can show either an upward trend or a downward trend, depending on the base year chosen.
- The same set of statistics can produce opposite conclusions at different levels of aggregation.
- You can always create a fraction by putting one variable upstairs and another variable downstairs, but that does not establish any causal relationship between them, nor does the resulting quotient have any necessary relationship to anything in the real world.

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[8] Cited in Howard Eves, *Return to Mathematical Circles* (Boston, 1988). Internet source: http://www-groups.dcs.st-and.ac.uk/~history/Quotations/Laplace.html

[9] For example, trying multiplying and dividing with Roman numerals.

[10] http://everydaymath.uchicago.edu/

[11] Thomas Sowell, *The Vision of the Anointed: Self-Congratulation as a Basis for Social Policy* (New York: Basic Books, 1995), pp. 102-103.

RESTORING THE THIRD "R"

BY JAMES D. NICKEL

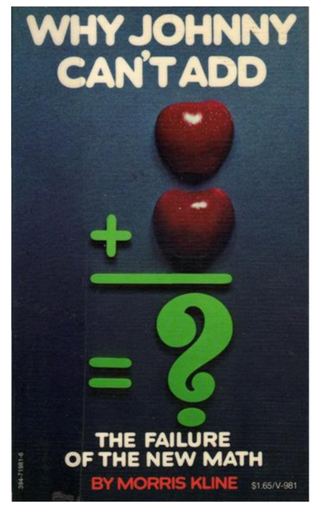
We cannot think rightly about the world you live in unless we have mastered the precepts of arithmetic. And, if you have mastered arithmetic, you better have the right set of thinking skills to go with this mastery.

> The problem with theories, even bad ones, is *that they can be implemented!*

RECENT TRENDS

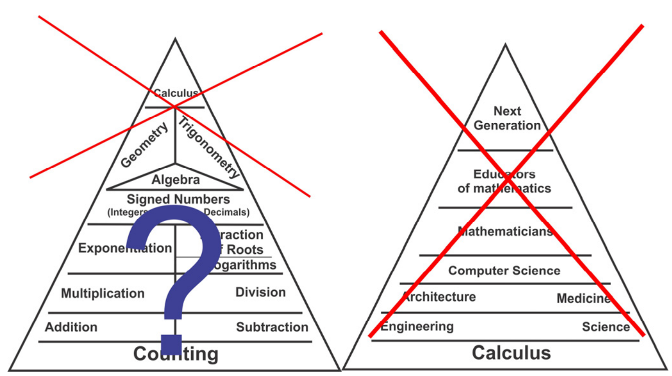
There has been a recent push in some government schools in America introduce algebra (i.e., generalized arithmetic) earlier than the traditional grade 9. The reasons given usually include one or all of the following:

- Math competency tests show that 15-year-olds in the United States trailed peers from 23 industrialized countries.
- Because of the global economy[12], more advanced math needs to be taught in high school.
- At stake, some politicians say, is the country's ability to produce enough scientists and engineers to compete in this global economy.



To those readers who remember the 1950s, this urgency should sound familiar. Because of Sputnik (1957), American leaders suddenly realized that the Russians were running ahead in the space race. To catch up, American educators implemented math programs without careful thought of the methods employed or timely measurement of the outcomes of those methods. The "New Math" was introduced, parodied by Tom Lehrer (1928-), American singer-songwriter, in a song about children who know that 2 + 3 = 3 + 2 but not that 2 + 3 = 5. In 1973, mathematician Morris Kline (1908-1992) proved that the new math emperor had no clothes in his ringing indictment entitled *Why Johnny Can't Add: The Failure of the New Math* (Vintage Books).[13]

What we have lost and are losing in these math programs, whether 50 years ago or today, is the instruction in the precepts of arithmetic. Unless students master arithmetic, algebra and subsequent courses are lost causes because, as we have seen, arithmetic, algebra and geometry are interconnected. Success in high school mathematics depends on success in arithmetic. If you do not get arithmetic right, you will be doing remedial work all the way up to college and note, parenthetically, how many remedial math classes (euphemism for arithmetic) are now being taught in college,



What these math programs aimed to accomplish generated, by their methods, the exact opposite. In spite of this, some students, for a variety of reasons, still manage to master mathematics. In contrast, consider the many students who fell through the cracks and are thereby missing in ac-

[12] See Thomas L. Friedman, *The World is Flat: A Brief History of the Twenty-first Century* (New York: Farrar, Straus and Giroux, [2005] 2006).

[13] This book, though dated, is still an excellent read.

tion. Consider the lost accomplishments, productivity, inventions, and innovations. Who can properly calculate the social, cultural, technological, and generational dynamics wrought by this loss?[14]

"FUZZY" MATH

To further illustrate the thesis that the problem with theories, even bad ones, is that they are implemented, let's explore two more recent examples: (1) the NCTM (National Council of Teachers of Mathematics) standards and (2) TERC (Technical Education Research Centers).

In 1989, NCTM, a nonprofit, nonpartisan education association with nearly 100,000 members and 250 affiliates in the United States and Canada, redefined the goals of mathematics education.[15] What is eye-opening about these goals is not what they state, *but what they do not state*. These goals do *not* include the teaching of rigor, algorithms, formal methods, logical reasoning, or calculation skills. According to the standards, the material is *not* the essence of mathematics instruction; the process of learning by self-discovery is the essence. A new phrase, *fuzzy* math, is now in vogue because of this self-discovery emphasis.[16]

Self-discovery learning is child-centric, not teacher-centric. The teacher, instead of teaching content, facilitates learning by encouraging discovery by investigation and trial and error. For example, the student learns about fractions by cutting sections of circles to try to discover relationships. If the cutting is inaccurate, the student might discover that $\frac{1}{4}+\frac{1}{3}=\frac{1}{2}$. Even though this answer is wrong (the sum is $\frac{7}{12}$), it is close enough to $\frac{1}{2}$ and is therefore correct regarding the process. Note how mathematics professors Philip J. Davis and Reuben Hersh grade this procedure:

> Some educators have soft-pedaled that [students produce right answers to math problems–JN]. In their view, the important thing is that the student hold the right methodological thoughts about the problem, not that the right answer is obtained. Try that one out at the fish market.[17]

Five years (1994) after implementing these standards, the State of California dropped to 48th place among the 50 American states in comparative mathematics tests. The percentage of students requiring remedial math in college increased by a factor of $2\frac{1}{2}$. In addition to these results, California high-tech companies reported that no local candidates were qualified.[18]

TERC, a not-for-profit education research and development organization dedicated to improving mathematics, science, and technology teaching and learning, is another example of what happens to mathematics instruction when standards are "fuzzy."[19] In TERC's K-5 Arithmetic program, the bias toward easy rather than hard multiplication facts is pervasive. The TERC curriculum also teaches multiplication by the trial and error method. Being a technical approach, the reason for the omission of the traditional algorithms for multiplication and division is because of

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[14] Likewise, no one can truly grasp the social dynamics created by the death via abortion of approximately 50,000,000 Americans due to the Roe v. Wade Supreme Court decision of 1973.

[15] See http://standards.nctm.org/ and http://www.nctm.org/

[16] The author recognizes that discovery, i.e., searching for numerical and spatial patterns, is an important tool in mathematics pedagogy, but it is *not* the only tool. Discovery, a fruit of investigating concrete problems (i.e., induction), should lead the student to rigor in algorithms and the use of deduction.

[17] Philip J. Davis and Reuben Hersh, *Descartes' Dream: The World According to Mathematics* (Boston: Houghton Mifflin, 1986), p. 154. Both process and answer must be right. To tell a student that all you want is the right process and that you do not care about the right answer could not only produce a fish market fiasco, but an engineer who uses the right process to construct a bridge that collapses due to a calculation error.

[18] Ron Aharoni, *Arithmetic for Parents: A Book for Grownups about Children's Mathematics* (El Cerrito, CA: Sumizdat, 2007), p. 194.

[19] See http://www.terc.edu/

the ever present calculator. This naiveté is breathtaking in its disregard of the reality that multiplication and division algorithms encapsulate fundamental computational and algebraic ideas and experiences necessary for future success in algebra and calculus.

It is no wonder that W. Stephen Wilson, mathematics professor at The Johns Hopkins University, had this to say about TERC in a letter published on 19 December 2008 in the Frederick County, Maryland daily newspaper:

> I am a research mathematician and it was never my intention to get involved in K-12 mathematics education. However, my son, who is now 17, started off his mathematics education using TERC Investigations, the program now used in Frederick County. I had no concept of how bad mathematics education could be. TERC Investigations was beyond my wildest imaginings. I have thoroughly reviewed the new TERC Investigations materials.
>
> They do not cover the necessary mathematics for a student to be successful (details can be found on my website, an easy Google away). More than 80 percent of the high school graduates in the U.S. go on to college. What that means is that the middle 5th grader will go to college and drop out, frequently because of their mathematics training. TERC Investigations prepares students do exactly that, drop out of college. I am writing this letter not to the newspaper or to the board or the teachers, but to the parents. If your child goes to a school that uses TERC Investigations, you should understand that it means your child's school has abdicated its responsibility to teach your child mathematics. By doing so, the responsibility now rests with the parents. Good luck. I'd be happy to talk to anyone about this.[20]

The fact is that we not only have "fuzzy" math, but "fuzzy" law, "fuzzy" history, "fuzzy" economics, "fuzzy" politicians, and a "fuzzy" citizenry should not, therefore, surprise us.[21]

WHY IS THIS HAPPENING?

Are these "fuzzy" mathematics programs a part of some integrated conspiracy? I doubt it. I believe that the teachers who are using these programs are sincere, but sincerity does not make one right. Dr. Glenn R. Martin (1935-2004), esteemed Professor History and Political Science at Indiana Wesleyan University, used to say, "We are either leaders or we are led; influenced or influential." These teachers are not leaders; most are being led without realizing what is happening. Led by what?

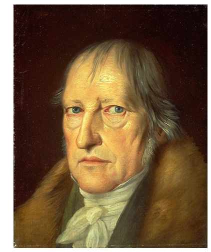
The fact is that we not only have "fuzzy" math, but "fuzzy" law, "fuzzy" history, "fuzzy" economics, "fuzzy" politicians, and a "fuzzy" citizenry should not therefore surprise us.

The American idea is grounded upon liberty and responsibility under God. The ideas of liberty and responsibility under God are the heritage of the Biblical Christian worldview. The American idea formed the backbone of early

[20] http://lizditz.typepad.com/i_speak_of_dreams/2008/12/i-am-a-mathematics-professor-at-the-johns-hopkins-university-in-baltimore-i-was-one-of-the-coauthors-of-the-fordham-foundati.html (retrieved 12 February 2009).

[21] In the United States, the Common Core standards were initiated ca. 2012. In mathematics, the emphasis of the standards is on reasoning and understanding which is right. When it comes to methodology, the standards sometimes overcomplicate matters. For example, the multiplication algorithm is reduced to an old Lattice method that complicates things by doing too much. The traditional right-to-left carry model is much easier, by comparison, because it simplifies things. The left-to-right no carry model that I introduce in my textbook, *The Dance of Number*, simplifies the traditional model making the algorithm's dance unfold higher to lower place-value answers in beautiful symmetry. It is the beauty of symmetry, simplification rather than over-complication, combined with understanding that builds genuine number sense in the student.

American education. Teachers once viewed schooling as the acquisition of a body of knowledge. They then challenged the student to use the scope of that knowledge to formulate a reasoned conclusion as a responsible individual. Daniel Fish's textbook was a by-product of this idea.



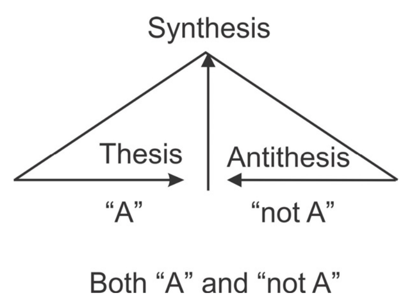
Georg Wilhelm Friedrich Hegel (Public Domain)

Everyday Mathematics, NCTM standards, and TERC are by-products of another idea. We can trace that idea to the German philosopher Georg Wilhelm Friedrich Hegel (1770-1831). Hegel's philosophy generated a significant revolution in the way people *think*. Hegel introduced synthetical thinking in contrast to antithetical thinking.

In brief, antithetical thinking is based upon absolutes. Either a statement, labeled A, is true or A is false.[22] Either $\frac{1}{4}+\frac{1}{3}=\frac{1}{2}$ or $\frac{1}{4}+\frac{1}{3}\neq\frac{1}{2}$. Antithetical thinking, being Biblical, comports with the nature of reality.

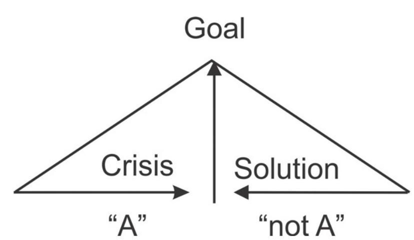
Synthetical thinking is based upon a denial of absolutes. Instead of "either A or not A" (thesis/antithesis) we have "both A and not A" (thesis/antithesis generates a synthesis).[23] Hence, it does not matter that $\frac{1}{4}+\frac{1}{3}=\frac{1}{2}$ or $\frac{1}{4}+\frac{1}{3}\neq\frac{1}{2}$; *what matters, the synthesis, is the process involved.*[24] Synthetical thinking does *not* comport with the nature of reality … ask any carpenter if $\frac{1}{4}+\frac{1}{3}=\frac{1}{2}$ comports with the reality of constructing a house.

Therefore, the approach to learning embraced by *Everyday Mathematics,* the NCTM standards, and TERC is Hegelian dialecticism. Promoters of these programs are being led by an idea antithetical to the *idea* that formed the basis for the freedoms and liberties that Americans have historically enjoyed.



Hegelian dialecticism also undergirds liberal and socialist agendas, plans dating back to the turn of the 20th century. These are the agendas that lay behind the "fuzzy" math programs. These socialistic goals bring clarity to what we are seeing, for example, in these pedagogical innovations. For the socialist, the purpose of education is to socialize the child, producing the collaborative, cooperative, collective child, a child that turns into an adult that can be easily manipulated and swayed, i.e., led, by the political pied pipers.[25]

Synthetical thinking is the way to achieve these goals.[26] The process is threefold. First, identify or create a crisis (e.g., poor math scores or "We are behind in the space race!"). This crisis is the thesis. Second, so-called experts appear and give theoretical and often unexamined solutions to the crisis at hand (e.g., pedagogical innovations like the "New Math" of the 1960s). This solution to



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[22] Technically, the statement must be in the form of a proposition. For example, God exists versus God does not exist.

[23] Philosophically, Hegelian thinking is called dialecticism, the juxtaposition or interaction of conflicting ideas.

[24] It is interesting to note that Hegelian dialecticism is the root of what is called "process philosophy," the ground of Darwinism, Pragmatism, Marxism, and Freudianism.

[25] John Dewey (1859-1952), American educator, emphasized the collectivist nature of education. The current political pied piper is the 44th President of the United States, Barack Hussein Obama II (1961-).

[26] See Thomas Sowell, *The Vision of the Anointed: Self-Congratulation as a Basis for Social Policy*, for more details.

the crisis is the antithesis. Third, these unexamined solutions are then implemented resulting in the synthesis; the goal in mind, the collective-minded citizenry, not mastery of arithmetic.[27]

The Elementary and Secondary Education Act (ESEA) of 2001, also known as "No Child Left Behind," illustrates this three-step process.[28] In the guise of helping American education, the goal of this act was to exert more federal control over the education system (the synthesis). First, the crisis; i.e., children are being "left behind!" The "experts" who constructed ESEA made it a tool to gain power by granting money to school districts on the proviso that they accept the strings attached. The accessibility of these federal dollars disestablished local support and control. ESEA also mandated that more socially related programs replace academic programs.

Second, ESEA implementation resulted, not only in a decrease in test scores but also in an increase in juvenile problems in children due to illiteracy and an ethical vaccum. In response to public and parental outcries, teachers, not the act or the Federal government, were held responsible. Attempts to fix the unforeseen problems created by this secondary crisis generated even more social programs. What was lost in these programs was that children still were not learning. Sound familiar?

Third, progressive socialist education, the goal, is now upon us. We are creating a generation of people, a "fuzzy" citizenry, incapable of thinking, reasoning, speaking and questioning. This new generation will be a ghost standing at the grave of its demise.

DIRECTION FOR REFORM

Irish statesman Edmund Burke (1729-1797) once said, "All that is necessary for the triumph of evil is that good men do nothing." It may appear that the night is coming down on Western civilization[29], but we need to be men and women who boldly act because we are stand on the edge of dawn. Let the resoluteness that Winston Churchill (1874-1965) offered the darkened world in 1940 ring afresh in our hearts, "Let this be our finest hour."

What should we do? First, remember what we do must be based upon information that is true. Gather the facts and be able to present reasoned alternatives to parents, educators, newspapers, boards, etc.

Teachers must be our priority because students are not the only ones who need instruction that is more rigorous. Too many elementary school teachers lack the expertise to teach arithmetic efficiently. It is a given that one cannot teach what one does not know, and students cannot love the subject unless teachers love the subject.[30] Most elementary school teachers have little exposure to high-level math in college, and they are more at home with words than numbers. Many fear math. Many do not understand the subtle nuances involved in teaching arithmetic let alone how to teach these nuances.

According to Israeli mathematician Ron Aharoni,

> ... elementary mathematics isn't simple at all. It has depth and beauty.... Proper teaching of mathematics depends more on an understanding of the mathematical principles than on educational tricks. It requires familiarity with the way the fine mathematical layers lie one upon the other.[31]

There are many profound and beautiful ideas embedded in arithmetical concepts. That is why "line upon line, precept upon precept" (Isaiah 28:10-13) instruction is required, not the haphazardness of self-discovery. The teaching of these ideas, layer upon layer, generates both mastery and appreciation. Aharoni remarks are apropos,

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[27] There is plenty of historical evidence to show that unexamined solutions, if implemented, created unforeseen problems. See Sowell for documentation.

[28] See http://www.k12.wa.us/esea/

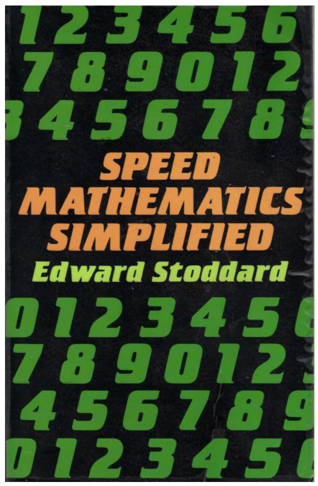
[29] The economic crisis that started in late 2008 may be the portent of such a cultural spiral into the void.

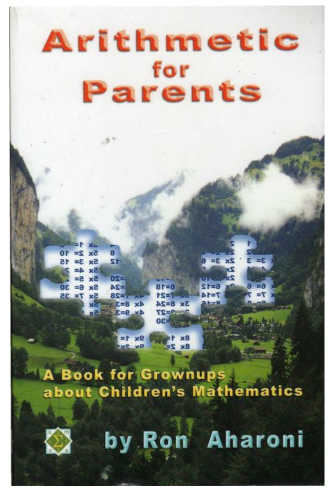
[30] Drawing an analogy with elementary reading instruction, would you want a teacher who has read "Dick and Jane" or would you want a teacher who loves reading and has read Shakespeare and other masters of English prose?

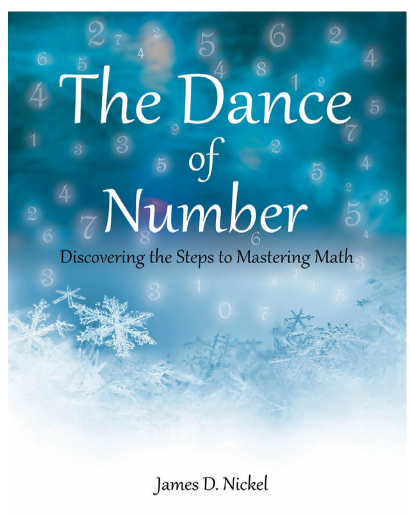
[31] Aharoni, pp. vi-vi.

> The elementary arithmetic we learned as children contains some of the most beautiful mathematical discoveries ever made. Why, then, is it not perceived by most people as beautiful? Mainly because it is often learned mechanically, in a way that does not reveal its beauty.[32]

Aharoni's words ought to entice you to buy his book (available from www.sumizdat.org) and, if you do, it would be well worth the price. Aharoni's explanation of the fine nuances of arithmetic pedagogy is the best available. Get two other books, Edward Stoddard's *Speed Mathematics Simplified* (http://store.doverpublications.com/)[33] Mastering all three will put you in the driver's seat of leadership when it comes to arithmetic education.







[32] *Ibid.*, p. 16.

[33] My forthcoming two-volume textbook, *The Dance of Number*, will incorporate the ideas of Aharoni, Stoddard, and a host of other resources, historical and scientific, accumulated as the result of nearly 40 years of research and teaching. See http://www.biblicalchristianworldview.net/news.html