Mathematics

A Nickel math maxim: It is better to go slow and master the material, than to go fast and get lost.

James Nickel

If nature leads us to mathematical forms of great beauty and simplicity … we cannot help think that they are true, that they reveal a genuine feature of nature.

Werner Heisenberg, *Physics and Beyond* (1971), p. 68.

Here was I, sitting at my desk for weeks on end, doing the most elaborate and sophisticated calculations to figure out how an electron should behave. And here was the electron on my little oil drop, knowing quite well how to behave without waiting for the result of my calculation. How could one seriously believe that the electron really cared about my calculation, one way or the other? And yet the experiments at Columbia showed that it did care. Somehow or other, all this complicated mathematics I was scribbling established rules that the electron on the oil drop was bound to follow. We know that this is so. Why it is so, why the electron pays attention to our mathematics, is a mystery even Einstein could not fathom.

Freeman Dyson, *Disturbing the Universe* (1981), p. 50.

Mathematics is the abstract key that turns the lock of the physical universe.

John Polkinghorne, *One World: The Interaction of Science and Theology (*1987), p. 46.

In the teaching and studying of mathematics, we need to develop "an appreciation of the beauty, extent, and vitality of mathematics."

Sherman K. Stein, *Mathematics: The Man-Made Universe* (1976), p. xiv.

The quest for beauty is the *ultimate* motivation of a mathematician, a pursuit that surprises most of us due to the way educators have generally been teaching math [There are glorious exceptions to this rule, but they are a tiny minority.]. The extent of mathematics advocates the vastness of its reach in the way it reveals the rationality of the universe, a rationality that points to a transcendent and personal *logos* as the ultimate ground thereof. The vitality of mathematics is that it grows, starting from a few root principles, it into the colossal, varied, intertwining, and magnificent branches of the mathematics tree.

It is an elemental truism that one cannot study theology aright unless one is forgiven and cleansed for only the pure in heart can see God (Matthew 5:8). The same purity holds for the right study and understanding of anything else. Even the study of mathematics is not an exception to this rule. Of course, evil people can learn how to do mathematical calculations and even derive proofs but the more profound your study, the more you peel off the layers of calculation and see the depths and beauty of genuine mathematics, the more you find that both your mind and heart are sanctified. This should not be a surprising consequence since the ground of mathematical beauty is God Himself.

James Nickel

"Mathematics is the music of reason."

James Joseph Sylvester (1814-1897)

"Mathematics is tricky ways of doing things that would be laborious otherwise."

Richard Feynman (1918-1988), lecture on the properties of photons at the University of Auckland (New Zealand), 1979

"Beauty without power [or mathematical utility] is futile. Power without beauty is liable to be impotent ... Both the pure artist [pure mathematician] and the pure bureaucrat [applied mathematician] are wrong or at least incomplete. If the teaching of mathematics had to be based on the theories of either one of them, that of the artist would do less harm. An artist [is] ... at least ... alive, and without life there is no growth. If such a man can teach children to love a subject for itself, there is always the hope that at a later age these children will turn their gifts to useful ends. But if they are left in the hands of the extreme utilitarians, they will have no gifts to turn."

Mathematics

Walter W. Sawyer (1911-2008)

Professor Jerry P. King, "The way to learn mathematics is to learn some fundamental parts of it truly and well. Once this is done, any other part of mathematics that must be learned can be learned." King demolishes the idea that to teach mathematics well you have to cover "millions" of topics. But, mathematics is not about topics per se, it is about fundamental principles. Hence, we teach more if we teach less, i.e., these fundamentals, truly and well.

James Nickel

Science is the study of the footprints of God in creation, His many and wonderful works (Psalm 104:24). Over the centuries, a language has been developed by man "imago Dei" as a commentary on the Speech of God who created and currently upholds all things, visible and invisible (John 1:1; Colossians 1:15-17). It is called Mathematics and its language is Algebra. To therefore trivialize Algebra as a "necessary evil" that students are required to do or to use textbooks that may teach computational skills, but not the language, is to diminish one's ability to think through and thereby penetrate creational glory. Being a language, Algebra has its own vocabulary, grammar (principles that govern its correct use), syntax (the part of grammar that concerns rules of word order), synonyms, negations, conventions, abbreviations, and sentence structure. To master this language is to learn how to read it (not just listen to a teacher explain it) and in reading it one learns to think mathematically and thereby obtain a sixth sense that enables one to see creation afresh in the glory of its God-given light.

James Nickel

Keith Devlin on the transition to university mathematics, "Even if they did well at math in school, most beginning university students are knocked off course for a while by the shift in emphasis, from the K-12 focus on mastering procedures to the 'mathematical thinking" characteristic of much university mathematics."
To make this transition easier, students need to be introduced to "mathematical thinking" as quickly as possible, including reinforcement, in their K-12 math experiences. The majority of K-12 textbooks on the market do little of that. I would expect little of this emphasis from K-8, but, for years 9-12, this must be done.

James Nickel

Symbols, formulas, functions, properties, axioms, postulates, derivations, theorems ... never stand alone. They are connected to reality: to people, to history, to culture, to utility, to beauty.

James Nickel

From Warren Esty, mathematics professor at Montana State University: "Regarding the future of mathematics teaching, all math teachers need to play a bit with wolframalpha.com. Put in your standard AP calculus questions and it will do them [wolfram apps on the iPad, iPod, and iPhone do the same]. You don't even have to know the notation. Just ask it, say, to differentiate or integrate anything: 'Integrate x cubed' or 'differentiate x cubed.' Then, when you have had it do several problems, ask yourself what is still worth teaching. Ask it 'Solve $2x + 3y = 10$ and $3x – y = 7$.' Make up some questions of your own. Then, think about the change in the world of mathematics. I am serious about this. Most of what we teach is not very valuable anymore. A machine can do it. What's left to teach? Concepts, language, connections. Understanding. How to formulate word problems. 'How to solve problems given in symbolism' is way down the list [of things required to know in today]! Learning to read math is now much more important than it used to be. If you have a math fact you need to know, wikipedia is often very good, if you can read it. So, I want to emphasize things worth knowing--things that come up a lot." The language of math is a language of syntax and symbols. If the fine nuances of its language are not mastered, then mathematics becomes a silent symphony and, ultimately, the Voice of God singing through it is no longer heard.

James Nickel

Mathematics

Just sighted on a bumper sticker: "4 out of 3 people have trouble with fractions."

James Nickel

In a class today (5 January 2012), we discussed the elliptical nature of the orbit of Halley's Comet. Its orbital period is 76 years. Mark Twain was born when it appeared 1835 and he died when it appeared in 1910. While living in Australia, I saw it clearly and distinctly with my young children in 1986. I told them that they would be in their 80s the "next time it came around" (2061). The class I taught today would be in their late 60s in 2061. Of course, I won't be around then so I told my class (as a homework assignment) to "remember the fun times with your Precalculus teacher" if you do see it in 2061. The regular patterns of God's heavenly clocks do "time stamp" one's life.

James Nickel

Albert Einstein argued before the Prussian Academy of Sciences 1921 (his speech is found in *Ideas and Opinions*, pp. 232-246) that geometry and experience are so closely intertwined that geometry must be regarded as a form of natural science. This means that geometry constitutes the epistemological foundation that lies at the depths of physics. Therefore, geometry cannot be isolated as an independent conceptual system complete and consistent on its own (and this is what happens far too often in the typical teaching of geometry). If we do this, geometry becomes merely an exercise in pure logic, and indeed, thereby, an empty and irrelevant one. The reality of the Incarnation demands that geometry be fleshed out into the real world.

James Nickel

Some well-meaning Christians have tried to use Bible "numerics" as the basis for mathematics. All have failed. The starting point is too weak because it fails to grasp the purpose of the Bible. Regarding mathematics, the purpose of the Bible is to provide foundational presuppositions about metaphysics, knowledge, and ethics. Man works from these, as part of the dominion mandate (naming is a tool of dominion and mathematics is about naming number and space), to develop the structure and discipline of mathematics, all the while understanding that the order, beauty, and symmetry is there as a *given*. Mathematics unfolds the coherence of thought and the universe, a coherence and rationality that "holds together" only because of the person of the Lord Jesus Christ (Col. 1:15-17).

James Nickel

Mastering mathematics is all about paying attention to the details. In the same manner, living the Christian life is all about being faithful in the little things. It is how we think and react when we are slighted, laughed at, kept waiting, treated rudely, hurt intentionally or unintentionally, crossed, contradicted, interrupted, or not preferred. In the sight of God and eternity's values, these are "little things," but they form the manner of our life, our likeness or unlikeness to God, i.e., our eternity.

James Nickel

A testament to the power of Maxwell's equations is all around us--radio, television, radar, wireless Internet, Bluetooth technology--all of this and more are rooted in electromagnetic field theory. These equations are not only beautiful, but are probably the most important equations in the history of science, mathematics, and ... mankind.

James Nickel

If a teacher doesn't see beauty in mathematics, he/she will not teach beauty. Hence, the method of instruction will become mere logical formalism, syllogisms that chatter, but do not compel.

James Nickel

Math texts tend toward this: "I have found an argument and I do not need to give you an understanding." We can "thank" both Euclid and Newton for this unfortunate heritage.

James Nickel

Mathematics

Sir Isaac Newton (1642-1727) developed a mathematical method (differential equations) that could both describe and predict dynamics (a universe in motion). He did not read these equations into this motion; these equations are tools by which we can understand variation, tools by which we can understand God's creation, a universe that reveals both constancy (in terms of law) and variation (in terms of motion).

James Nickel

In its history, mathematics, as a language, has resolutely pointed to a given order, a specificity of order, in the universe. Naturalistic mathematicians and scientists have done their best to avoid the implications of such order; i.e., the revelation of consistent rationality in the universe points to an ultimate ground of rationality beyond it.

James Nickel

The beauty, symmetry, and wonder revealed in the four electromagnetic equations derived by James Clerk Maxwell is a portal through which one, who has eyes to see, can catch a faint glimpse, an infinitesimal glimmer, of the beauty and wonder of the Author and Sustainer of electromagnetism.

James Nickel

One has to be blind to beauty to not be able to see it in a mathematical proof (e.g., Euclid's proof of the infinity of the primes). As Morris Kline once said, "An elegantly executed proof is a poem in all but the form in which it is written."

James Nickel

Yesterday (11 February 2011), I was showing a class two ways of multiplying 38 times 21 and I got two answers! Umm! I did it again and same result! I was calculating a sub-product, 8 x 2 = 10! Ouch (multiply, Mr. Teacher, don't add)! I tried to give an excuse (not enough sleep, old brains, etc.) to which one student replied, "In the military, we are told to say, 'No excuses, Sir!'" I stood corrected!

James Nickel

Yesterday (11 February 2011), I showed two ways to solve a math problem. The first took 10 minutes (an approximate answer); the second took 5 seconds (exact answer). One student replied, "Math rocks!" Another student said, not so enthusiastically, "I will never forgive you for taking us through the 10 minute path." I said, "You have to; Jesus commands you to forgive me." The whole class got a good laugh on that one.

James Nickel

A quote from Karl Weierstrass (a 19th century mathematician who worked on deriving the definition of a limit of a function, a definition essential to the understanding of calculus). This definition is usually taught to first-year calculus students but fewer than 2% probably understand it. He said, "No mathematician can be a complete mathematician unless he is also something of a poet."

James Nickel

… if you begin with an autonomous rationality, what you come to is mathematics (that which can be measured), and mathematics only deals with particulars, not universals. Therefore you never get beyond mechanics.

Francis Schaeffer (1912-1984)

To do mathematics without paying attention to the context of creation (the physical world) and history is to replace the Biblical Christian view of mathematics with a Platonic view of mathematics; i.e., the absolutization of the abstract or "ideals."

James Nickel

From Professor David Bressoud on Advanced Placement math classes in high school, "... AP courses into a high school will not turn around students whose PreK-8 education has not adequately prepared them."

Mathematics

In other words, mastery of arithmetic is the foundation for success in Algebra, Geometry, Trigonometry, and Calculus.

James Nickel

"Apropos: Please forget everything you have learned in [high - JN] school; for you haven't learned it." Edmund Landau, "Foundations of Analysis: The Arithmetic of Whole, Rational, Irrational and Complex Numbers," (Chelsea, [1951] 1960), near the conclusion of his "Preface for the Student."

James Nickel

Mathematical sophistication is independent of technical background. Background refers to how much mathematics you know. Sophistication refers to how deeply you know it.

Jerry P. King, *Mathematics in 10 Lessons: The Grand Tour*, p. 82

A mathematical poem from one of my math students: "I used to think math was no fun ... cause I didn't know how it was done ... then Euler my hero told me that zero equals *e* to the *i* times pi plus one."

James Nickel

Math joke for today: Mr. City Slicker visited Mr. Farmer and asked how many cows Mr. Farmer had in his pasture. Mr. Farmer replied instantly, "174." "How did you figure that out so fast?" asked Mr. City Slicker. "It was easy," replied Mr. Farmer, "I just counted the number of legs and divided by four."

James Nickel

Having "had" a subject in school is not much of an advantage if you don't remember anything!

Warren W. Esty, Instructor's Manual for Precalculus (4th edition)

A circle is a happy thing to be–
Think how the joyful perpendicular
Erected at the kiss of tangency
Must meet my central point, my avator.
And lovely as I am, yet only 3
Points are needed to determine me.

Christopher Morley (1890-1957)

A very considerable part of the reputation mathematics has for being difficult is due to its being taught from the outside, as a memory task, rather than an exercise in understanding and insight.

Walter Warwick Sawyer (1911-2008), *Vision in Elementary Mathematics*, p. 4

No mathematician can be a complete mathematician unless he is also something of a poet.

Karl Weierstrass (1815-1897)

Mathematics is the loom upon which God weaves the fabric of the universe.

Clifford A. Pickover, *The Loom of God: Mathematical Tapestries at the Edge of Time*, p. 16

Where is the Biblical God in math? Mathematics is man's way to order his numerical and spatial thought structures. Since man is made in God's image, then man's mind has a "6th sense" regarding numbers and space. In other words, man can think numerically and spatially because he is *imago Dei*. From Genesis 1, we see that God is the author of space (the heavens and the earth). Space and number are in unity (one reveals the other). Hence, because of the spatially and numerical patterns revealed in creation (made in, by, through, and for Christ, the *logos* of God) are orderly (as a reflection of God's creational covenants; cf. Genesis 8:22) and man has an orderly mind (*imago Deo*), then mathematics, which is man's search for patterns in the created order (mind investigating matter), is reflective of God's covenantal order founded in the Agency of Creation, the Lord Jesus Christ (Genesis 1, John 1:1-3; Colossians 1:15-17), the Ultimacy in rationality, order, harmony, symmetry, proportion, balance, and interconnectedness.

James Nickel

Mathematics

The possibility of an applied mathematics is an expression, in terms of natural science, of the Christian belief that nature is the creation of an omnipotent God. This belief is what replaced the Greek conception of nature as the realm of imprecision with the Renaissance conception of nature as the realm of precision. The Platonism of Renaissance natural science is not fundamentally Platonic, it is fundamentally Christian. Christian thought is adapting Platonism to its own ends, or begetting upon Platonism an idea which Platonism proper would never have originated or even tolerated …. Christianity, by maintaining that God is omnipotent and that the world of nature is a world of God's creating, completely altered the situation. It became a matter of faith that the world of nature should be regarded no longer as the realm of imprecision, but as the realm of precision …. Galileo, the true father of modern science, restated the Pythagorean-Platonic standpoint in his own words by proclaiming that the book of nature is a book written by God in the language of mathematics …. Galileo is deliberately applying to nature the principle which Augustine laid down with regard to the Holy Scriptures, the book *par excellence* "written by the hand of God"; that whatever doubts may arise about the meaning of this or that passage, it has a meaning, and the meaning is true (*Confessions,* XII, 23-4).

R. G. Collingwood, *An Essay on Metaphysics* (1940), pp. 253-256

Now you may ask, "What is mathematics doing in a physics lecture?" We have several possible excuses: first, of course, mathematics is an important tool, but that would only excuse us for giving the formula [related to the study of oscillatory systems - JN] in two minutes. On the other hand, in theoretical physics we discover that all our laws can be written in mathematical form; and that this has a certain simplicity and beauty about it. So, ultimately, in order to understand nature it may be necessary to have a deeper understanding of mathematical relationships. But the real reason is that the subject is enjoyable ...

Richard P. Feynman, *Lectures on Physics,* 1:22-1

Mathematics is as much an art form as is art and music. It deserves to be taught by teachers who love it, just as music teachers love music and art teachers love art.

Calvin C. Clawson, *Mathematical Sorcery: Revealing the Secrets of Numbers*, p. 2

Ultimately, we are faced with the question of why - why does mathematics work in the material universe? ... Why does mathematics work so magnificently as a model to explain our universe? Scientists use mathematical models of the physical world to make claims and predictions about the world. Why should this relationship between model and physical reality exist unless there is some underlying connection? If numbers are only objects of thought, then why are they so wonderfully useful in analyzing the material universe?

Calvin C. Clawson, *Mathematical Mysteries: The Beauty and Magic of Numbers*, p. 52

The single most compelling reason to explore the world of mathematics is that it is beautiful, and pondering its intriguing ideas is great fun.

Calvin C. Clawson, *Mathematical Mysteries: The Beauty and Magic of Numbers*, p. 2

God "arranged everything according to measure, number, and weight" (Wisdom 11:20) and ... he made man in His image, endowing him thereby with a mental ability to see precisely this arrangement.

Stanley L. Jaki, *Numbers Decide and Other Essays*, p. 39

You must realize that a mastery of mathematics involves both levels [computational and theoretical], although your tastes may direct you more strongly to one or the other, or both ... Be warned that deficiency at either level can ultimately hinder you in your work. Independently of need, however, it should be a source of pleasure to understand why a mathematical result is true, i.e. to understand its proof as well as to understand how to use the result in concrete circumstances.

Serge Lange, *Basic Mathematics*, p. xi

Indeed, only a few are mathematically gifted in the sense that they are endowed with the talent to discover

new mathematical facts. But by the same token, only very few are muscially gifted in that they are able to compose music. Nevertheless there are many who can understand and perhaps reproduce music, or who at least enjoy it. We believe that the number of people who can understand simple mathematical ideas is not relatively smaller than the number of those who are commonly called musical, and that their interest will be stimulated if only we can eliminate the aversion toward mathematics that so many have acquired from childhood experiences.

Hans Rademacher and Otto Toeplitz, *The Enjoyment of Mathematics*, p. 5

I believe that "beauty" is not a test at all, but simply the result when true mathematics is discovered.

Larry L. Zimmerman, *Truth and the Transcendent*, p. 12

To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature.

Richard P. Feynman, *The Character of Physical Law*, p. 40

There is also a rhythm and a pattern between the phenomena of nature which is not apparent to the eye, but only to the eye of analysis; and it is these rhythms and patterns which we call Physical Laws.

Richard P. Feynman, *The Character of Physical Law*, p. 13

God created everything (visible and invisible) in, through, and by Christ, the *logos* (John 1:1-3; Colossians 1:15-17). All of the mathematical patterns, numerical or spatial, in the creation therefore find their ultimate source of interconnectedness, order, beauty, and meaning in the second person of the Trinity.

James Nickel

Very few people appreciate more than some elementary aspects of mathematical beauty, much of it revealing itself only to mathematicians in the study and creation of intricately crafted proofs, barely within the reach of the most highly trained human minds. As a mathematician, I declare that I have established the truth of a theorem by writing at the end of its proof the three letters Q. E. D., an abbreviation for the Latin phrase *quod erat demonstrandum*, which translates as "what had to be proved." On the one hand, Q. E. D. is a synonym for truth and beauty in mathematics; on the other hand, it represents the seemingly inaccessible side of this ancient science.

Burkard Polster, Australian mathematician, *Q. E. D. Beauty in Mathematical Proof*, p. 1

It is difficult to give an idea of the vast scope of modern mathematics. The word "scope" is not the best; I have in mind an expanse swarming with beautiful details, not the uniform expanse of a bare plain, but a region of a beautiful country, first seen from a distance, but worthy of being surveyed from one end to the other and studied even in it smallest details: its valleys, streams, rocks, woods and flowers.

Arthur Cayley (1821-1895), English mathematician, cited in *Calculus Gems*, by George F. Simmons, p. 215

It is often said that mathematics should be studied for its usefulness. This is quite right. It should be studied for the philosophic insight it affords, and more so for the logic it uses and creates. But perhaps its main claim to your attention is based on the intrinsic beauty it reveals to those who can see it. The poet said: "Beauty is its own excuse for being." I would add to this: The cultivation of beauty is its own reward.

Nathan A. Court, *Mathematics in Fun and in Earnest*, p. 122

Now we've come to something really spooky. The applied mathematics process *works* ... Time and again, the applied mathematics process demonstrates the unreasonable effectiveness of mathematics in the natural sciences ... How can this be? The miracle itself is wonder enough. Wigner wrote: "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve." Yes. But if, as I claim, the motivation for the development of mathematics is primarily aesthetic, and not utilitarian, then the wonder compounds itself. We are talking about the *paradox of the utility of beauty*. And we are dealing with a miracle of second order of magnitude.

Mathematics

Jerry P. King, *The Art of Mathematics*, p. 121

... pure mathematics is analysis in the mathematical world; applied mathematics is pure mathematics which has a "pre-image" in the real world.

Jerry P. King, *The Art of Mathematics*, p. 287

There can be no evasion of mathematical methods in the long run if one is to penetrate to the real heart of physics, for mathematics is the key with which physics has unlocked one gate after another of nature's strongholds, which have been inaccessible by all other means.

Lloyd W. Taylor, *Physics: The Pioneer Science*, 1:33.