# Note on the Study of Mathematics

## by Alfred North Whitehead (1861-1947)

Alfred North Whitehead, born in Ramsgate, Kent on 15 February 1861, was taught by his father, an Anglican clergyman, until the age of 14. He was then sent to Sherborne School, Dorset, one of the best schools in Great Britain. He showed a special gift for mathematics and was a natural leader. In 1880, he entered Trinity College, Cambridge, on a scholarship and attended only lectures in mathematics (his interests in literature, religion, and philosophy were nourished only by conversation). He did well in the honors examination of 1883-1884, won a Trinity Fellowship, and was appointed the staff as a lecturer in mathematics. The subject of his fellowship dissertation was James Maxwell's theory of electromagnetism. In 1891, he married Evelyn Willoughby Wade, born in France, a child of impoverished Irish landed gentry, and educated in a convent. Her sense for beauty and drama enriched his life immensely.

Shortly before his marriage and through his contact with Cardinal John Henry Newman (1801-1890), Whitehead, in spite of his Anglican upbringing and heritage, began to consider the tenets of Roman Catholicism. He studied theology for eight years and then officially became an "agnostic." He sold his theological library and although his agnosticism did not survive World War I (his son, Eric, was killed in action in 1918), he never again was a member of any church.

In 1891, Whitehead wrote *Treatise on Modern Algebra.* While working on the second volume (between 1898 and 1903), he abandoned it to begin a joint and ambitious project with Bertrand Russell (1872-1970), one of his brilliant students. Both began developing a comprehensive case to found all of mathematics on the basis of formal logic (called to logicism school of mathematical foundations). The result was a three-volume tome (written almost entirely in mathematical symbols) entitled *Principia Mathematica* (1910-1913). The development of incompletness theorems (i.e., mathematics cannot be founded in logic or axiomatics alone) by the logician and mathematician Kurt Gödel (1906-1973) brought their ambitious goal to an immediate halt in 1931.

In 1903, Whitehead was appointed senior lecturer at Trinity College. His interests in mathematics remained mostly philosophical in terms of grasping its nature and unifying ideas. Because he never made any profound and new discoveries in mathematics, there was little prospect of him becoming a professor in mathematics at Cambridge. He decided to leave Trinity (with a good pension) and move to London. There he wrote one of his first books aimed at a wider audience entitled *An Introduction to Mathematics* (1911). Nearly a century later, this book is still considered one of the best of its kind. The excerpt below is taken from the Bibliography of the Oxford paperback version (1958), pp. 187-188.

Whitehead went on to write many books on education and philosophy. He wrote much on how to improve mathematics education. He stressed teaching the unity of the subject. To him, the teaching of mathematics should not consist of giving the student a pedantic set of of disconnected and unfathomable exercises. He saw that the teaching of mathematics in his era was dry, mechanical,

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fragmented and lacking in aethestic beauty and appreciation. He found that most teachers of his day were handcuffed by a system of uniform examinations set by outside examiners. His prophetic plea was to help educators understand that education was not to pack scraps of disassociated information into the students, but to stimulate reflective intellectual development. Although his ideas had little affect on changing British practice, they inspired many teachers in England, the United States, and elsewhere.

In 1924, he moved to America to take a teaching position (professor of philosophy) at Harvard University. In 1925, he gave a series of lectures in Boston which were published in the book *Science and the Modern World*. In these lectures, he stunned the distinguished audience of scholars by stating that modern science owed its foundation to the medieval insistence upon the rationality of God.

He gave his remaining years to writing (primarily on philosophy) and teaching (including lecturing at many US campuses). His teaching at Harvard endeared students to him and most students found attendance at his lectures a memorable experience. They would flock to his "Sunday evenings," where he opened his home to them for informal discussions. Harvard did not retire him until 1937 when he was 76. He loved Americans but always remained a true Englishman. A Fellow of the Royal Society since 1903, he was elected to the British Academy in 1931. In 1945, he received the Order of Merit. After his death in 1947, his body was cremated and there was no funeral. As he directed his widow, all his unpublished manuscripts and private correspondence were destroyed.

The difficulty that beginners find in the study of this science is due to the large amount of technical detail which has been allowed to accumulate in the elementary text-books, obscuring the important ideas.

The first subjects of study, apart from a knowledge of arithmetic which is presupposed, must be elementary geometry and elementary algebra. The courses in both should be short, giving only the necessary ideas; the algebra should be studied graphically, so that in practice the ideas of elementary co-ordinate geometry are also being assimilated. The next pair of subjects should be elementary trigonometry and the co-ordinate geometry of the straight line and circle. The latter subject is a short one; for it really merges into the algebra. The student is then prepared to enter upon conic sections, a very short course of geometrical conic sections and a longer one of analytical conics. But in all these courses great care should be taken not to overload the mind with more detail then is necessary for the exemplification of the fundamental ideas.

> ... the unfortunate learner finds himself struggling to acquire a knowledge of a mass of details which are not illuminated by any general conception. Without a doubt, technical

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> facility is a first requisite for valuable mental activity: we shall fail to appreciate the rhythm of Milton, or the passion of Shelley, so long as we find it necessary to spell the words and are not quite certain of the forms of the individual letters. In this sense there is no royal road to learning. But it is equally an error to confine attention to technical processes, excluding consideration of general ideas. Here lies the road to pedantry.
> Alfred North Whitehead, *An Introduction to Mathematics,* pp. 2-3.

The differential calculus and afterwards the integral calculus now remain to be attacked on the same system. A good teacher will already have illustrated them by the consideration of special cases in the course on algebra and co-ordinate geometry. Some short book on three-dimensional geometry must be also read.

> The progress of science consists in observing these interconnexions and in showing with a patient ingenuity that the events of this ever-shifting world are but examples of a few general connexions or relations called laws. To see what is general in which is particular and what is permanent in what is transitory is the aim of scientific thought.
> Alfred North Whitehead, *An Introduction to Mathematics*, p. 4.

The elementary course of mathematics is sufficient for some types of professional career. It is also the necessary preliminary for any one wishing to study the subject for its intrinsic interest. He is now prepared to commence on a more extended course. He must not, however, hope to be able to master it as a whole. The science has grown to such vast proportions that probably no living mathematician can claim to have achieved this.

> One of the most fascinating characteristics of mathematics is the surprising way in which the ideas and results of different parts of the subject dovetail into each other. During the discussions of this and the previous chapter [imaginary numbers - JN] we have been guided merely by the most abstract of pure mathematical considerations; and yet at the end of them we have been led back to the most fundamental of all the laws of nature, laws which have to be in the mind of every engineer as he designs an engine and of every naval architect as he calculates the stability of a ship. It is no paradox to say that in our most theoretical moods we may

be nearest to our most practical applications.
Alfred North Whitehead, *An Introduction to Mathematics*, p. 71.

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Whitehead went on to mention four "treatises" on the subject of mathematics to be read *after* this preliminary course. I know for certain that one of these books, by G. H. Hardy, is still in print.

1. Oswald Veblen and John Wesley. Young, *Projective Geometry*. 2 vols. Boston: Ginn and Co., 1910, 1918.
2. A. A. Albert, *Introduction to Algebraic Theories.* Chicago: University of Chicago Press, 1941.
3. H. Lamb, *An Elementary Course of Infinitesimal Calculus.* Cambridge: University Press, 1924.
4. Godfrey H. Hardy, *A Course in Pure Mathematics*. Cambridge: University Press, 1944.