LOGARITHMS AND LARGE NUMBERS

BY JAMES D. NICKEL

> The heavens declare the glory [the inescapable weight of the godhood] of God (Psalm 19:1).

Historically, logarithms were invented at the same time that scientists like Johannes Kepler (1571-1630) needed a method to ease the calculation (multiplication and division) of very large numbers. These numbers were astronomical in that they were connected to *astronomy*, i.e., the law of the stars. This law (or patterned order) revealed in the vast outreaches of the heavens (what we call interstellar space) reflects the covenantal faithfulness of God (read Jeremiah 31:35 and Genesis 1:14-18).

John Napier (Public Domain)

Symbolic algebra and logarithms, along with decimal notation[1], decimal fractions[2], and trigonometry[3], all combined to produce a powerful mathematical and scientific synergy in the 17th century.

Early in that century, the Scottish mathematician John Napier (1550-1617) invented logarithms. He was a very interesting man. In mathematics, he simplified the writing of decimal fractions by advocating the use of a decimal point to separate the whole part of a number from its fractional part. He also developed a mechanical calculating device, called "Napier's bones." You could solve multiplication and division problems using this apparatus. He once caught pigeons who were eating the grain in his yard by soaking peas in brandy and thereby making the birds drunk. Napier once suspected one of his servants of theft. He brought all of his servants together and announced that he had a black rooster that would identify the guilty party. He ordered his servants into a dark room and asked each to pat the rooster on its back. Unknown to them, Napier had coated the bird's back with a layer of chimney soot. Upon leaving the room, each servant had to give a "show of hands." The guilty one, fearing to touch the rooster, came out "clean handed" thus sealing his culpability. Yes, as Napier predicted, his rooster did identify the thief.

Note Napier's own analysis of the situation in 1614:

> Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers which involve a tedious expenditure of time, as well as being subject to 'slippery errors'.... I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.[4]

In 1914 at Edinburgh, Lord Moulton, in his inaugural address commemorating the 300th anniversary of this strategic invention, paid Napier this tribute:

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[1] Decimal notation is a way of representing numbers by 10 symbols (i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) where position "counts." For example, 305 stands for 3 hundreds, zero tens and 5 ones. 35 stands for 3 tens and 5 ones.

[2] Decimal fractions express the ratio of two numbers (e.g., 2/5) in terms of decimal expansion (e.g., 2/5 = 0.4 where 4 represents the tenths position).

[3] Trigonometry means "measure of three angles." It is a branch of mathematics that uses the ratios of the sides of a right-angled (90 degree) triangle to assist in measuring unknown distances (e.g., the distance from the Earth to the Moon).

[4] Cited in George A. Gibson, "Napier and the Invention of Logarithms," *Handbook of the Napier Tercentenary Celebration, or Modern Instruments and Methods of Calculation*, ed. E. M. Horsburgh (Los Angeles: Tomash Publishers, [1914] 1982), p. 9.

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The invention of logarithms came on the world as a bolt from the blue. No previous work had led up to it, foreshadowed it or heralded its arrival. It stands isolated, breaking in upon human thought abruptly without borrowing from other work of intellects or following known lines of mathematical thought.[5]

One of the "genius" effects of logarithms is that they turn a multiplication problem into an addition problem and a division problem into a subtraction problem. Logarithms make use of the product law and quotient law of exponents.

Product law: $\left(b^x\right)\left(b^y\right) = b^{x+y}$ (*b* is called the base). A multiplication problem is turned into the addition of exponents.

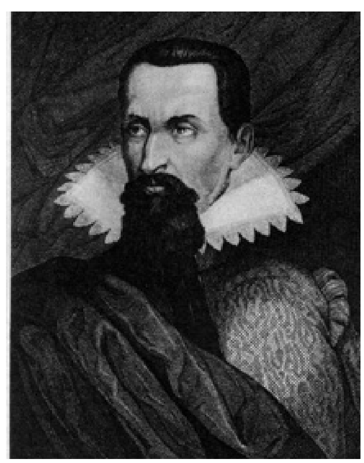
Quotient law: $\frac{b^x}{b^y} = b^{x-y}$. A division problem is turned into the subtraction of exponents.

Napier and his fellow co-workers in science and mathematics converted these laws into logarithms (recognizing the simple equivalency that *logs are exponents*): $b^x = c \Leftrightarrow \log_b c = x$. Using logarithms, the product and quotient laws become:

Product law: $\log_b\left(jk\right) = \log_b j + \log_b k$ (again, *b* is the base)

Quotient law: $\log_b\left(\frac{j}{k}\right) = \log_b j - \log_b k$

Johannes Kepler (1571-1630) prepared extensive logarithmic tables to help him calculate astronomical distances. It is a truism that the invention of logarithms helped fuel the Scientific Revolution of the 16th century. Back then, scientists, astronomers especially, spent huge amounts of time crunching numbers on paper. By cutting the time they spent doing arithmetic, logarithms effectively gave them a longer and more productive scientific life.



Johannes Kepler (Public Domain)

When I was a student (before the age of electronic calculators), logarithmic tables were found in the back of my mathematics textbooks. With the advent of electronic calculators, these tables have disappeared because we no longer need them to multiply and divide large numbers. It is interesting to note that in spite of the obsolete nature of logarithmic tables, our modern calculators still have a *log* and *ln* keys.[6] Why?

Although logarithms are no longer necessary as a calculation device, their properties are still very useful. In the words of mathematics professor Eli Maor, "But if logarithms have lost their role as the centerpiece of computational mathematics, the logarithmic *function* remains central to almost every branch of mathematics, pure or applied. It shows up in a host of applications, ranging from physics and chemistry to biology, psychology, art, and music."[7]

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[5]Inaugural address, "The Invention of Logarithms," *Napier Tercentenary Memorial Volume*, ed. Cargill Gilston Knott (London: Longmans, Green, and Company, 1915), p. 3. Napier published his invention in 1614 in a work entitled *Mirifici logarithmorum canonis descriptio* (Description of the wonderful canon of logarithms).

[6] The *log* key stands for logarithms to the base 10 or "common logarithms" and the *ln* key stands for logarithms to the base *e* (≈ 2.71818) or "natural logarithms."

[7] Eli Maor, *e: The Story of a Number* (Princeton: Princeton University Press, 1994), p. 16.

Kepler used logarithms to "scope the heavens." Marveling at the grandeur of the skies, his prayer is a fitting conclusion to this essay:

> "Accordingly let this do for our *envoi* concerning the work of God the Creator. It now remains that at last, with my eyes and hands removed from the tablet of demonstrations and lifted up towards the heavens, I should pray, devout and supplicating, to the Father of lights: O Thou Who dost by the light of nature promote in us the desire for the light of grace, that by its means Thou mayest transport us into the light of glory, I give thanks to Thee, O Lord Creator, Who hast delighted me with Thy makings and in the works of Thy hands have I exulted. Behold! now, I have completed that work of my profession, having employed as much power of mind as Thou didst give to me; to the men who are going to read those demonstrations I have made manifest the glory of Thy works, as much of its infinity as the narrows of my intellect could apprehend."[8]

[8] Johannes Kepler, *Epitome of Copernican Astronomy & Harmonies of the World*, trans. Charles Glenn Wallis (Amherst: Prometheus Books, [1618-1621, 1939] 1995), p. 240.