Light … Be!

Electromagnetism is a fundamental force in the universe. It is the underlying reason for the way things work in the micro and macro realms. It is essential in applied technology (from microwave ovens to electronic watches to personal computers). Even our bodies, from electrochemical nerve impulses to the electric signals controlling our heartbeat, can be explained using its principles.

James Clerk Maxwell (Public Domain)

There is a magnificent wonder in electromagnetism. The Scottish physicist James Clerk Maxwell (1831-1879) was the first scientist to explain, in terms of mathematics, their symmetry and beauty. In the early 1860s, when Maxwell had just turned 30 years of age, he published the four equations that (1) describe electricity and magnetism separately and then (2) shows the connection between electric and magnetic force fields.[1] In his day, Maxwell already knew the first two equations, for they were developed by and named after the German mathematician Carl Friedrich Gauss (1777-1855). Maxwell also knew, based upon the work of the English physicist and ist Michael Faraday (1791-1867), that a moving magnetic field produces electricity (this is called Faraday's Law of Induction). This phenomenon is the basis for many modern technologies, including computer discs (CDs), audio and videotapes, and, most importantly, electric power generators.[2]

> One factor that has remained constant through all the twists and turns of the history of physical science is the decisive importance of the mathematical imagination.
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> *Freeman J. Dyson*, "Mathematics in the Physical Sciences," *Scientific American*, September 1964.

When looking at the situation as it existed in the middle of the 19th century, Maxwell surmised, given the presupposition that creation reveals the beauty of symmetry, that if moving magnetic fields produce electric fields, then why can't it go the other way? He incorporated this conjecture into what are now

> Every one of our laws is a purely mathematical statement in rather complex and abstruse mathematics … Why? I have not the slightest idea … To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature.
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> Richard Feynman, *The Character of Physical Law*, p. 39, 58.

[1] Fields, in physics, represent the invisible interactions that exert forces on electric charges and magnetic poles. You can "make visible" the invisible magnetic field, in terms of its "lines of force," by sprinkling iron filings on a magnet.
[2] To move the magnetic field, these electric generators can use steam, coal, wind, water, or natural gas.

called the laws of electromagnetism. These laws consist of four equations, beautiful and symmetric equations of power, which describe all electromagnetic phenomena (sans quantum physics).

To explain the derivation and context for these laws would be too much for an essay of this scope.[3] I will state two ways of writing them in mathematical form and then explain what is being revealed by them. Since two of the four equations involve the physics of change (magnetism to electricity and vice versa), the calculus is the perfect, even predetermined, tool for their derivation and expression.

Here are the two forms, represented using calculus inverses, i.e., differentials and integrals.

Name	**Equation**		**Meaning**
	Differential Form	**Integral Form**	
Gauss's Law of Electricity	$\nabla \bullet E = \dfrac{\rho}{\varepsilon_0}$	$\oint E \bullet dA = \dfrac{q}{\varepsilon_0}$	How electric charges produce an electric field or how charges attract and repel each other (electric field lines begin and end on charges).
Gauss's Law of Magnetism	$\nabla \bullet B = 0$	$\oint B \bullet dA = 0$	No magnetic charge or no isolated magnetic poles; i.e., they always come in North/South pairs (magnetic field lines do *not* begin or end).
Maxwell-Faraday Law of Induction	$\nabla \times E = -\dfrac{\partial B}{\partial t}$	$\oint E \bullet dl = -\dfrac{d\phi_B}{dt}$	Changing a magnetic flux (strength of a field force in a given area) produces an electric field.
Maxwell's Conjecture	$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \dfrac{\partial E}{\partial t}$	$\oint B \bullet dl = \mu_0 I$	Electric current (i.e., changing electricity) produces a magnetic field.

Briefly described, these symbols stand for:

E is the electric field.
B is the magnetic field.
J is the total current density.
ε_0 is a universal constant of electricity (in farads per meter).

[3] See Richard Wolfson and Jay M. Pasachoff, *Physics for Scientists and Engineers* (Reading: Addison-Wesley, 1999), Chapters 23 to 34.

ρ is the total charge density.

μ_0 is a universal constant of magnetism (in newtons per ampere squared).

t is time.

$\nabla \bullet$ is the divergent operator per meter acting on either E or B. It is a vector operator that measures the magnitude of a vector field's source or sink at a given point, in terms of a signed scalar.

$\nabla \times$ is the curl operator per meter acting on either E or B. It is a vector operator that describes the infinitesimal rotation of a three-dimensional vector field.

$\frac{\partial}{\partial t}$ is a partial derivative.[4]

dl where l is the letter ℓ. It is the differential vector element of *path length* tangential to the path/curve represented by l.

dA is the differential vector element of *surface area* represented by A.

$\oint \mathbf{E} \bullet d\mathbf{l}$ or $\oint \mathbf{B} \bullet d\mathbf{l}$ represents integration of E or B over a closed curve.

q is the net electric charge.

ϕ_B is the magnetic flux through any surface (not necessarily closed).

I is the net electric current passing through any surface.

After deriving these equations, Maxwell first noticed their beauty and symmetry. Every mathematician/scientist since has responded in awe to this revelation. It is as if these equations are a portal revealing the deep inner workings of the very foundations of the invisible world of electromagnetism.

From these equations, Maxwell predicted the existence of electromagnetic waves. These waves are structural combinations of electric and magnetic fields that travel through empty space. He then calculated the speed of these waves; i.e., approximately 1 foot per nanosecond[5] or 186,000 miles/second … *the speed of light!*

Hence, to Maxwell, light must be an electromagnetic wave. Thus, optical science was now united to electromagnetism. There are other electromagnetic waves and they all travel at the speed of light but differ by their vibration frequencies (measured in Hertz, or Hz = one cycle/sec) or by their wavelengths (λ). Trigonometric functions serve to describe these waves mathematically.

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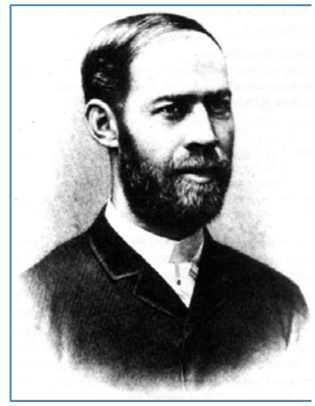
[4] A partial derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant (as opposed to the total derivative, in which all variables are allowed to vary).

[5] A nanosecond is a *billionth* of a second or 1 ns = 10^{-9} s.

<table>
<tr><th colspan="7">The Electromagnetic Spectrum</th></tr>
<tr><th>f (Hz)</th><th>λ (meters)</th><td rowspan="2"></td><td rowspan="5">Gamma
Rays
↓</td><td rowspan="4"></td><td rowspan="7"></td><td rowspan="7"></td></tr>
<tr><td>10^{21}</td><td>3×10^{-13}</td></tr>
<tr><td>10^{20}</td><td>3×10^{-12}</td><td rowspan="5">↑
X-rays
↓</td></tr>
<tr><td>10^{19}</td><td>3×10^{-11}</td></tr>
<tr><td>10^{18}</td><td>3×10^{-10}</td><td rowspan="3">↑
Ultraviolet
radiation
↓</td></tr>
<tr><td>10^{17}</td><td>3×10^{-9}</td><td rowspan="3"></td></tr>
<tr><td>10^{16}</td><td>3×10^{-8}</td></tr>
<tr><td>10^{15}</td><td>3×10^{-7}</td><td rowspan="5"></td><td>Visible
light</td><td>Violet 4000Å
Indigo 4300Å
Blue 4800Å
Green 5300Å
Yellow 5800Å
Orange 6100Å
Red 7000Å</td><td>4×10^{-7} m

7×10^{-7} m</td></tr>
<tr><td>10^{14}</td><td>3×10^{-6}</td><td rowspan="3">↑
Infrared
radiation
↓</td><td rowspan="6"></td><td rowspan="3"></td><td></td></tr>
<tr><td>10^{13}</td><td>3×10^{-5}</td><td rowspan="5">↑
Micro-
waves
↓</td></tr>
<tr><td>10^{12}</td><td>3×10^{-4}`</td></tr>
<tr><td>10^{11}</td><td>3×10^{-3}</td><td rowspan="3"></td><td rowspan="9">↑
Communications
bands:
Amateur, police,
airplanes, etc.
↓</td></tr>
<tr><td>10^{10}</td><td>3×10^{-2}</td><td rowspan="11">↑
Radio
waves
↓</td></tr>
<tr><td>10^{9}</td><td>3×10^{-1}</td></tr>
<tr><td>10^{8}</td><td>3×10^{0}</td><td rowspan="2">↑
TV
↓</td><td rowspan="4">↑
AM/FM
radio
↓</td><td rowspan="9"></td></tr>
<tr><td>10^{7}</td><td>3×10^{1}</td></tr>
<tr><td>10^{6}</td><td>3×10^{2}</td><td rowspan="7"></td></tr>
<tr><td>10^{5}</td><td>3×10^{3}</td></tr>
<tr><td>10^{4}</td><td>3×10^{4}</td><td rowspan="5"></td></tr>
<tr><td>10^{3}</td><td>3×10^{5}</td></tr>
<tr><td>10^{2}</td><td>3×10^{6}</td><td rowspan="3"></td></tr>
<tr><td>10^{1}</td><td>3×10^{7}</td></tr>
<tr><td>10^{0}</td><td>3×10^{8}</td></tr>
</table>

Speaking of Hz, it is named to honor the German physicist Heinrich Hertz (1857-1894). In 1887, he confirmed Maxwell's equations by generating and receiving electromagnetic waves in his laboratory. In 1901, the Italian inventor Guglielmo Marconi (1874-1937) transmitted radio waves across the Atlantic Ocean.



Heinrich Hertz (Public Domain)

The invisible nature of reality pulsates with vibrations. In Genesis 1:3, we read that God spoke light into existence.

"Light … Be!"

"And, light was!"

God's Word spoke light into existence and, according to John 1:1-3, the Word of Creation, the person through whom all things were made, is the second person of the Trinity, the Lord Jesus Christ. Jesus Christ is the ultimate source of rationality (the *logos*) of the universe. This same Jesus holds together or sustains everything in creation, things visible and invisible (Colossians 1:15-17). Hence, without the creative and sustaining power of God in Christ, Maxwell could never have developed his set of equations to describe the invisible nature of electromagnetic spectrum.

The beauty, symmetry, and wonder revealed in the four electromagnetic equations derived by James Clerk Maxwell is a portal through which one, who has eyes to see, can catch a faint glimpse, an infinitesimal glimmer, of the beauty and wonder of the Author and Sustainer of electromagnetism.

> One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.
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> Heinrich Hertz, cited in E. T Bell, *Men of Mathematics*, p. 16.