HOW TO FORGET THE MULTIPLICATION TABLE

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There are eleven basic rules that govern all work in elementary algebra. Five of these rules relate to addition, five relate to multiplication, and the final rule connects addition to multiplication. These rules encapsulate all of the basic properties of real numbers. Because the structure of the set of real numbers reflects all of these rules, mathematicians call the real numbers a *field*.[1] Here they are in symbolic form:

> Note: This essay is extracted from a Lesson from the forthcoming textbook *Mathematics: Building on Foundations*.

1. $a, b \in \mathbb{R} \Rightarrow a + b \in \mathbb{R}$. ($\Rightarrow$ means "implies.")
2. $\forall a, b \in \mathbb{R}, a + b = b + a$. ($\forall$ means "for all" or "for every.")
3. $\forall a$, b, and $c \in \mathbb{R}, a + (b + c) = (a + b) + c$.
4. $\forall a \in \mathbb{R}, \exists$ an element in $\mathbb{R}$ called $0 \ni 0 + a = a$. Remember, $\ni$ means "such that" and $\exists$ means "there exists."
5. For each pair a and $b \in \mathbb{R}$, there is exactly one $x \in \mathbb{R} \ni a + x = b$.
6. $a, b \in \mathbb{R} \Rightarrow ab \in \mathbb{R}$.
7. $\forall a, b \in \mathbb{R}, ab = ba$.
8. $\forall a$, b, and $c \in \mathbb{R}, a(bc) = (ab)c$.
9. $\forall a \in \mathbb{R}, \exists$ an element in $\mathbb{R}$ called $1 \ni 1a = a$.
10. For each pair a and $b \in \mathbb{R}$ (where $a \neq 0$), there is exactly one $x \in \mathbb{R} \ni ax = b$.
11. $\forall a$, b, and $c \in \mathbb{R}, a(b + c) = ab + ac$.

The closure property for real numbers under addition and multiplication connect to their respective inverses. The inverse operation of addition is subtraction and the inverse operation of multiplication is division. One inverse operation of exponentiation is extraction of roots. For a review, the following table illustrates these operations and their inverses.

Operation	*Inverse*
Addition ($7 + 3 = 10$)	Subtraction ($10 - 3 = 7$)
Multiplication ($6 \times 5 = 30$)	Division ($30/5 = 6$)
Exponentiation ($2^2 = 4$)	Extraction of roots ($\sqrt{4} = \pm 2$)

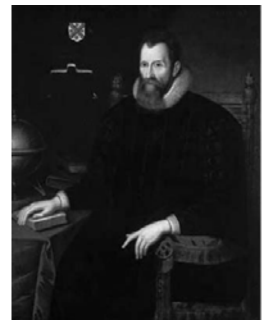
The inverse of raising any number x to the second power, i.e., $x^2 = a$, is called extracting the square (from 2) root, i.e., $\sqrt{a} = \pm x$. The inverse of raising any number x to the third power, i.e., $x^3 = a$, is called extracting the cube (from 3) root, i.e., $\sqrt[3]{a} = x$. The inverse of raising any number to the fourth power, i.e., $x^4 = a$, is called extraction the fourth root i.e., $\sqrt[4]{a} = \pm x$. In symbols, these operations look as follows:

Raising to the n^{th} power	*Extracting the n^{th} root*
$2^2 = 4$	$\sqrt{4} = \pm 2$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$2^4 = 16$	$\sqrt[4]{16} = \pm 2$
$2^n = b$	$\sqrt[n]{b} = \pm 2$ if n is even

[1] There is actually a distinction between what mathematicians call a field and an ordered field, but we will not embrace such minutiae in this essay.

Raising to the n^{th} power	*Extracting the n^{th} root*
	$\sqrt[n]{b} = 2$ if n is odd

Before we continue, note the odd/even principle. When you are extracting the n^{th} root of a *positive* number and *n* is *even*, you will always get two answers, one positive and the other negative. This is because a negative number times a negative number equals a positive number; i.e., (-2)(-2) = 4. Note also that (-2)(-2)(-2)(-2) = 16. If you are multiplying an *even* number of negative numbers, your answer will also be positive. If you are multiplying an *odd* number of negative numbers, the answer will always be negative. For example, (-2)(-2)(-2) = -8 but (2)(2)(2) = 8. Therefore, $\sqrt[3]{8} \neq -2$, but $\sqrt[3]{8} = 2$.


John Napier (Public Domain)

Before the advent of the electronic hand held calculator (in the late 1960s), all arithmetical operations had to be worked out by hand. By the early 17^{th} century, great advances had been made both in astronomy and in exploring the world through sea voyages. Both these advances necessitated performing arithmetical calculations using large numbers. A Scottish mathematician, John Napier (1550-1617) noted these difficulties and echoed the bane of many students of arithmetic,

> Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplication, divisions, square and cubical extractions of great numbers…. I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.[2]

Napier removed these difficulties by developing a system of "arithmetic" that replaced multiplication by addition and division by subtraction. Consider the table below (powers of 2):

n	1	2	3	4	5	6	7	8	9	10	11	12
2^n	2	4	8	16	32	64	128	256	512	1024	2048	4096

Napier first noted that if he multiplied any number in row 2 by any other number in row 2, his answer was a number in row 2. Second, he noted that when you *multiplied* any two numbers in row 2, then the answer correlated to an *addition* problem in row 1 (he made use of the *sum law of exponents*). Inversely, if you *divided* any two numbers in row 2, then the answer correlated to a *subtraction* problem in row 1 (the inverse of the sum law of exponents).

Row 2	*Row 1*
$2 \times 4 = 8$	1 + 2 = 3 (read 8 in row 2)
$2 \times 8 = 16$	1 + 3 = 4 (read 16 in row 2)
$8 \times 16 = 128$	3 + 4 = 7 (read 128 in row 2)
$16 \times 64 = 1024$	4 + 6 = 10 (read 1,024 in row 2)
$\frac{2048}{16} = 128$	11 – 4 = 7 (read 128 in row 2)
$\frac{64}{8} = 8$	6 – 3 = 3 (read 8 in row 2)

By this method, Napier arrived at *another inverse* of exponentiation. Given $2^x = n$, recall that we denote 2 as the *base* and *x* as the *exponent*. In exponentiation, we are given the base and the exponent. From this, we

[2] John Napier, *Mirifici logarithmorum canonis description* (1614). Cited in George A. Gibson, "Napier and the Invention of Logarithms," in *Handbook of the Napier Tercentenary Celebration, or Modern Instruments and Methods of Calculations,* ed. E. M. Horsburgh (Los Angeles: Tomash Publishers, [1914] 1982), p. 9.

determine that result; i.e., n. For example, $2^4 = 16$. In Napier's inverse, we are given the result (i.e., n) and the *base* i.e., 2). From this, we determine the *exponent* (i.e., x). He called this process "finding the *logarithm*."[3] For example, if $2^x = 16$, then x (the logarithm of 16) = 4. The logarithmic table looks as follows:

Number	*Logarithm*
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1024	10
2048	11
4096	12

This table has one serious limitation. What happens if we want to multiply 3 by 22? What is the logarithm of 3? What is the logarithm of 22? From the table, we note that the logarithm of 3 must be between 1 and 2 and the logarithm of 22 must be between 4 and 5. To find the logarithm of 3, we must determine x such that $2^x = 3$. We know for certain that x will not be a positive integer; it will be a *fraction* or its decimal equivalent.

Fractions do cause some inconvenience in calculations. Our table does not yet contain any fractions (the smallest is $2^0 = 1$). We must now introduce powers of 2 that are less than 1 if we want to express fractions in our table. Hence, we must consider 2^{-1}, 2^{-2}, 2^{-3}, etc. When we raise any number to a negative exponent, it means you take its reciprocal. In symbols, $a^{-n} = \frac{1}{a^n}$. Hence, $2^{-1} = \frac{1}{2} = 0.5$, $2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$, and $2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$. We can add these numbers to our table:

Number	*Logarithm*
0.0625	-4
0.125	-3
0.25	-2
0.5	-1
1	0
2	1
4	2
8	3

[3] Logarithm is derived from two Greek words, *logos* (reason or proportion) and *arithmos* (number), and it literally means "proportion number." Although, we introduced exponents (Lesson 2.5) before logarithms, according to mathematics historian Howard Eves (1911-2004), "One of the anomalies in the history of mathematics is the fact that logarithms were discovered before exponents were in use." See Howard Eves, *An Introduction to the History of Mathematics* (New York: Holt, Rhinehart and Winston, [1953, 1964, 1969] 1976, p. 250.

This helps us somewhat with finding the logarithm of numbers less than 1. But, there are still gaps in the table. How do we find the logarithm of 3; i.e. determine *x* such that $2^x = 3$? As noted before, $1 < x < 2$ (that is, *x* must be between 1 and 2). *x*, the logarithm of 3, must be a *fractional exponent*. We must be able to determine 2^x where $x = \frac{a}{b}$.

Let's consider how to calculate 2^x when $x = 1\frac{1}{2}$. We can rewrite $1\frac{1}{2}$ as $\frac{3}{2}$. We now consider $2^{\frac{3}{2}}$. Let's square this number and apply the *product law of exponents*.

$$\left(2^{\frac{3}{2}}\right)^2 = 2^{\frac{3}{2}\times 2} = 2^{\frac{6}{2}} = 2^3 = 8. \text{ Since } \left(2^{\frac{3}{2}}\right)^2 = 8, \text{then } 2^{\frac{3}{2}} = \sqrt{8} \approx 2.8$$

Note that $\sqrt{8}$ is an irrational number and that we rounded it off to the nearest tenth. Since $\frac{3}{2} = 1.5$, then we can add another entry to our table:

Number	*Logarithm*
2	1
2.8	1.5
4	2

We haven't quite got the logarithm of 3, but we are getting close. It can be shown that the solution to $2^x = 3$ is an irrational number (*x* cannot be written as a ratio of two integers) and that we can approximate *x* to any desired precision using fractional powers.

The process that I have led you through is exactly the pathway that Napier took in constructing his logarithm tables (it took him *twenty* years to do it in the pre-computer and pre-calculator days). In our example, we are constructing logarithms to the base 2. This is signified as follows: $\log_2 8 = 3$ or, in general, $\log_b x = y$. The inverse operation is signified as $2^3 = 8$ or, in general, $b^y = x$. Hence we have this equivalency:

> The miraculous powers of modern calculation are due to three inventions: the Arabic Notation, Decimal Fractions, and Logarithms.
>
> Florian Cajori, *A History of Mathematics* (1893).

$$b^y = x \Leftrightarrow \log_b x = y$$

With this equivalency, we can technically note the two inverses of exponentiation:

Exponentiation	*Inverse*	
$b^x = y$	Computing logarithms	$x = \log_b y$
$x^b = y$	Extraction of roots	$x = \sqrt[b]{y} = y^{1/b}$

The list below enumerates all the essential properties of logarithms (for now, just take note of the third and fourth property; we shall prove them as an exercise):

1. Addition property: $\log_b (xy) = \log_b x + \log_b y$
2. Subtraction property: $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

3. Power property: $\log_b x^n = n\log_b x$
4. Root extraction property: $\log_b \sqrt[n]{x} = \left(\frac{1}{n}\right)\log_b x$

Until the advent of the hand held calculator, most high school and college textbooks contained logarithmic tables as appendices. The base commonly used was 10 and it was signified without a subscript. It was called the *common logarithm*.[4] If you have a scientific calculator, the common logarithm key is signified by *log*. For example, log 100 = 2. Try it out with your calculator. The inverse operation is $10^2 = 100$. Here is a table of logarithms to the base 10:

Number	*Logarithm to the base 10*
0.0001	-4
0.001	-3
0.01	-2
0.1	-1
1	0
10	1
100	2
1000	3
10,000	4

Note the much longer gaps in the table between the numbers. log 1 = 0 and log 10 = 1. In contrast, for logarithms to the base 2, $\log_2 1 = 0$ and $\log_2 2 = 1$. It will take more effort to fill in the gaps for logarithms to the base 10. Note another key next to the *log* key on your scientific calculator. It is signified by *ln*. This logarithm is called the *natural logarithm*. The base of the natural logarithms is a very famous and significant number that Leonhard Euler denoted by the letter *e*.[5] *e* is an irrational number and it is approximately equal to 2.718281828 …. Why would this number be used as a base?

John Napier came very close to discovering this number. As far as historians of mathematics can ascertain, *e* first appeared in history in the context of financial calculations. *Deposit that thought in your mental bank for a moment*. We shall withdraw it after we explore the construction of natural logarithms.

We noted that 10 is not a very good base of logarithms because of the large gaps. 2 is a better base but it still has many gaps. We can try 1 as the base but that really does no good since all powers of 1 are 1. It is not good to try a base less than 1 because a fraction less than 1 raised to a power gets smaller and smaller. Take note of successive powers of $\frac{1}{2}$ in the table below:

[4] The English mathematician Henry Briggs (1561-1630) developed the first base 10 logarithmic tables. Common logarithms are sometimes called Briggsian logarithms in his honor.

[5] Euler may have chosen this letter to stand for *exponential*. Since the letters a, b, c, and d were already frequently used in mathematics, Euler may have just choose the next available letter in the alphabet to represent this unusual number. It is unlikely that Euler choose this letter because of his name. As a Christian (his father was a Calvinist pastor), Euler understood and practiced the Christian grace of humility.

n	$\left(\frac{1}{2}\right)^n$
1	$\frac{1}{2} = 0.5$
2	$\frac{1}{4} = 0.25$
3	$\frac{1}{8} = 0.125$
4	$\frac{1}{16} = 0.0625$
5	$\frac{1}{32} = 0.03125$

So, let's try a number between 1 and 2 as a base, say 1.1. Note the successive powers of 1.1 in the table below. There is a connection to Pascal's triangle here. Do you see it?

n	1.1^n
0	1
1	1.1
2	1.21
3	1.331
4	1.4641
5	1.61051
6	1.771561
7	1.948717
8	2.14358881

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Pascal's Triangle

Note immediately that these numbers grow slowly and that there are many numbers between 1 and 2 before we try to calculate those troublesome gaps. It seems that choosing a smaller number would be an even better choice. Let's try 1.001. In this case, we separate the terms in Pascal's triangle by pairs of zeroes (note how the terms fit neatly into the decimal expansion):

n	1.001^n
0	1
1	1.001
2	1.002001
3	1.0030003001
4	1.004006004001
5	1.005010010005001
6	1.006015020015006001
7	1.007021035035021007001
8	1.008028056070056028008001

At first we may wonder if we will ever reach 2. Eventually we will, yet very slowly. Mathematicians have proven that the powers of any number greater than 1 (even slightly greater than 1) will converge to infinity. The closer the number is to 1, the slower the rate of growth.

Although this table contains an excellent density of numbers, the drawback is its slow rate of growth. *Very large exponents are needed to produce small numbers*. For example, $1.001^{1000} \approx 2.716923932$. Take note of this number (Hint: compare it with the numerical value of *e* as we defined it earlier in this essay). So, if the exponents were 1,000 times as big, we could make the table more proportionate. So, let's consider 1.001^{1000} as a base. Now, we are going to raise this base to a fractional power as follows and make use of the *product law of exponents:*

$$\left(1.001^{1000}\right)^{\frac{1}{1000}} = 1.001^{1000\times\frac{1}{1000}} = 1.001^{\frac{1000}{1000}} = 1.001^{1}$$

$$\left(1.001^{1000}\right)^{\frac{2}{1000}} = 1.001^{1000\times\frac{2}{1000}} = 1.001^{\frac{2000}{1000}} = 1.001^{2}$$

$$\left(1.001^{1000}\right)^{\frac{3}{1000}} = 1.001^{1000\times\frac{3}{1000}} = 1.001^{\frac{3000}{1000}} = 1.001^{3}$$

etc.

Note that we are successively raising 1.001^{1000} to a fractional power in steps of $\frac{1}{1000}$. Converting these fractions to decimals, we get:

$\frac{1}{1000}$	0.001
$\frac{2}{1000}$	0.002
$\frac{3}{1000}$	0.003
$\frac{4}{1000}$	0.004
etc.	

Note also that we get the same results as before. Inspect the table below:

$(1.001^{1000})^{0}$	1.001^{0}	1
$(1.001^{1000})^{0.001}$	1.001^{1}	1.001
$(1.001^{1000})^{0.002}$	1.001^{2}	1.002001
$(1.001^{1000})^{0.003}$	1.001^{3}	1.0030003001
$(1.001^{1000})^{0.004}$	1.001^{4}	1.004006004001
$(1.001^{1000})^{0.005}$	1.001^{5}	1.005010010005001
$(1.001^{1000})^{0.006}$	1.001^{6}	1.006015020015006001
$(1.001^{1000})^{0.007}$	1.001^{7}	1.007021035035021007001
$(1.001^{1000})^{0.008}$	1.001^{8}	1.008028056070056028008001

Note the pattern that has developed. The exponents and the corresponding results *do not grow disproportionately*. Yet, the density is unimpaired. Therefore, the base of 1.001^{1000} is an excellent choice. Can we improve on this choice? Yes. Consider the base of 1.0001^{10000}. Why stop there? What about the base of 1.00001^{100000}? We could go "on and on" *ad infinitum* with this type of procedure until we converge to the *natural* or best number. What is this number? It is *e*, the base of the *natural* logarithms. Note the table below:

1.001^{1000}	2.716923932
1.0001^{10000}	2.718145927
1.00001^{100000}	2.718268237
$1.000001^{1000000}$	2.718280469
$1.0000001^{10000000}$	2.718281693
$1.00000001^{100000000}$	2.718281815
e	2.718281828

Let's withdraw our previously deposited financial thought from the bank. What does *e* have to do with money? Central to the concept of money is the concept of interest. Interest is what banks charge you for borrowing money from them. If you do not have the money to buy an item and the bank does, then the bank loans you the purchasing power to buy the item (this is called the principal). In return for this loan, the bank expects a repayment of this principal over time. An extra amount is charged to you for the use of this principal. This amount is called interest. It is a way to "pay the bank back" for its loss of purchasing power due to its loan to you.

The Bible presupposes *time* and *interest* in its teaching about restitution (see Exodus 22:1-15; Leviticus 6:1-7; Numbers 5:5-8; Luke 19:1-9). Restitution means to "make amends or to recompense an injury." The Bible teaches two forms of restitution (between man and God and between man and man). Man's sin against God is atoned for by the blood of Jesus Christ (man is "made right" with God through faith in the person and work of Jesus Christ). Man's sin against man is "made right" via restitution. In the case of theft, the Bible requires *payment of interest* for the stolen goods (cf. Leviticus 6:4-5; Exodus 22:1). This interest payment recognizes that when a man steals another man's goods, he is stealing another man's *capital.* By robbing man of his capital, a thief is depriving man of any increase on that capital during the time that it is in the thief's possession. Not only must the property be restored by a thief, a thief must also *restore its value over time.* In the same sense (minus the "theft" concept), this is what banks do when they charge interest for money borrowed.

There are two types of interest, simple and compound. Both are governed by mathematical formulas. The *simple interest* formula for deriving how much you must pay a bank back (called the maturity value) for a loan is:

$$S = P(1 + rt)$$

where S = maturity value, P = the principal (amount of the loan), t = interest rate per time period, and t = length of time you take to repay the principal. Note that *r* and *t* must be in the same time units (e.g., interest *per year* with a repayment schedule of *x years*). Suppose you borrow \$1000 at 12% per year $\left(\frac{12}{100} = 0.12\right)$ interest for 3 years. What must you pay back?

$$S = 1000[1 + 0.12(3)]$$
$$S = 1000\,(1.36) = 1360$$

The maturity value is \$1360. Normally, with a bank, you pay the loan back on a monthly basis. Your payments would be \$37.78 per month. In three years you would pay back the \$1000 plus an interest charge of \$360 for using the money.

Suppose that the interest rate is 1% $\left(\frac{1}{100} = 0.01\right)$ per month and your payment schedule is 36 months (3 years). What is S, the maturity value then?

$$S = 1000[1 + 0.01(36)]$$
$$S = 1000(1.36) = 1360$$

The maturity value would be the same.

Compound interest is governed by this formula:

$$S = P(1 + r)^t$$

where, as before, S = maturity value, P = the principal, r = interest rate per time period, and t = length of time you take to repay the principal. Let's now consider the same situation: a loan of $1000 at 12% interest per year with a repayment schedule of 3 years. What is S?

$$S = 1000(1 + 0.12)^3$$
$$S = 1000\ (1.12)^3 = 1404.93$$

Note the increase in the maturity value from $1360 calculated at simple interest to $1404.93 calculated at compound interest ($39.03 per month repayment). What is the increase? $44.93. Let's now increase the loan repayment to 15 years. What happens then? S, under simple interest is:

$$S = 1000\ [1 + 0.12(15)]$$
$$S = 1000\ (2.8) = 2800$$

It *pays* to pay off a loan early! Although your monthly payments are only $15.56 a month $\left(\frac{2800}{180}\right)$, you are paying *$1800 interest* on a $1000 loan. No wonder why, in Scripture, a loan for emergency needs was limited to *6 years* (see Deuteronomy 15:1-6).

The situation is much worse with compound interest.

$$S = 1000(1 + 0.12)^{15}$$
$$S = 1000(1.12)^{15} = 5473.57$$

In this case your monthly payments are $30.41 a month $\left(\frac{5473.57}{180}\right)$. But, you are paying *$4473.57 interest* on a $1000 loan. Ouch! The lesson to be learned about borrowing money: *the higher the interest rate and the longer the repayment schedule, the more you pay in interest.*

Some banks compute compounded interest not once but several times a year. If an annual interest rate of 12% is compounded semiannually, the bank will use one-half of the annual interest rate as the rate per period. Hence, in one year a principal of $1000 will be compounded twice, each time at a rate of 6%. This will amount to 1000×1.06^2 or $1123.60, about $3.60 cents more than the same principal would yield if compounded annually at 12%.

The banking industry uses all kinds of compounding periods – from annual to semiannual to quarterly to monthly to weekly to even *daily*. Suppose the compounding is done *n* times a year. For each period, the bank uses the annual interest divided by *n*; i.e., $\frac{r}{n}$. Since in *t* years there are *nt* periods, the formula for compound interest becomes:

$$S = P\left(1+\frac{r}{n}\right)^{nt}$$

If n = 1 (compounded annually), the formula returns to its original state, $S = P(1 + r)^t$. Let's compare what happens to a $1000 loan at 12% compounded for different periods.

Period	nt	$\frac{r}{n}$	S
Annually	1	0.12	$1120.00
Semiannually	2	0.06	$1123.60
Quarterly	4	0.03	$1125.51
Monthly	12	0.01	$1126.83
Weekly	52	0.0023077	$1127.34
Daily	365	0.0003288	$1127.49

$7.49 is the extra that a bank receives from compounding daily as against compounding annually. It hardly makes a difference how the interest is compounded. In this case, the difference lies in the principal. The larger the principal, the larger the interest charged on a daily basis. Try the formula for the above periods with a principal of $1,000,000 and note what happens. Note that the reverse of borrowing money is investing money. Investing money at compound interest generates, over the long term, significant earnings. What bites you one way (borrowing) rewards you the other way (investing).

Let's now consider a hypothetical case. Let's assume that the bank loans money at an interest rate of 100%. Of course, no one in his or her right mind would take out a loan at this rate. And, if you could find an investment vehicle that returns 100% you would probably "jump on it." Let's consider what happens in this case. Instead of a principal of $1000, let P = 1 and t = 1. Our equation now becomes:

$$S = P\left(1+\frac{1}{n}\right)^{n}$$

Let's see what happens as we vary *n*.

n	$1+\frac{1}{n}$	S
1	2	2
2	1.5	2.25
3	1.333…	2.37037
4	1.25	2.44141
5	1.2	2.48832
10	1.1	2.59374
50	1.02	2.69159
100	1.01	2.70481
1,000	1.001	2.71692
10,0000	1.0001	2.71815
100,000	1.00001	2.71827
1,000,000	1.000001	2.71828
10,000,000	1.0000001	2.71828

Lo and behold, the maturity value approaches *e*, the base of the natural logarithms. Mathematicians have signified this relationship as follows:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \approx 2.71828$$

Using rhetorical algebra, these symbols are read as follows: The limit as *n* approaches infinity (or increases without bound) of the sequence of numbers defined by $\left(1+\frac{1}{n}\right)^n$ converges to *e*.

This is almost too incredible to comprehend. Yet, it is another wonder of mathematics. The marvelous connection between the base of the natural logarithms and the calculation of compound interest at 100% is too amazing to be just a coincidence. Is it not one of the astounding particular details in the Creator's grand design?