ALGEBRA: WHAT'S IT ALL ABOUT?

BY JAMES D. NICKEL

Algebra is a fearful word for most high school students and for far too many adults. Algebra: What is it? What's it all about? Before I attempt to answer these questions, let's do a brief study in history, etymology, and world view analysis.

Algebra is an Arabic word. In the 7th century AD, the followers of Mohammed began ransacking the nations of North Africa, the Middle East, and Europe. In spite of some destruction of knowledge and books, Muslim leaders generally respected and absorbed the manuscripts of the ancient Greek mathematicians. They were also responsible for introducing to Europe several valuable concepts that they borrowed from the Hindus of India: the base 10 decimal system, positional notation,[1] the number zero, and negative numbers. The decimal system of writing numbers (e.g., 1, 2, 3, 4, etc.) proved to be much more efficient than the old, cumbersome Greek letters or Roman numerals. This method of number writing acted as an indispensable asset in later developments of European mathematics and science.

Beginning in the 7th century, the Hindus began working with the number zero, negative numbers, and the idea of positional notation. All three of these concepts were foreign to ancient Greek mathematics.

> Note: This essay is extracted from a set of Lessons from the forthcoming textbook *Mathematics: The Language of Science*, a text that assumes successful completion of a one year course in elementary algebra.

Arabic mathematicians not only enhanced the rudiments of Algebra (initiated by the ancient Greeks); they also gave it the name. In 830 AD, the astronomer Mohammed ibn Musa al-Khowarizmi (ca. 780-ca. 850), working in Baghdad, wrote a book titled *Hisab al-jabr w'al-muqabalah*, which literally means "science of reunion and opposition" or more freely "science of transposition and cancellation."[2] Al-jabr, used in a non-mathematical sense, was something a barber did in those times. A barber not only cut hair, he mended bones. Thus, an algebrista was a bonesetter.[3] In Galatians 6:2, the Apostle Paul states, "Brethren, if a man be overtaken in a fault, ye which are spiritual *restore* such a one in the spirit of meekness, considering thyself lest thou also be tempted." The Greek word for restore means "to join back together" (as in setting a broken bone) or "to mend" (as in repairing torn nets). The Greek word for restore means exactly the same as the Arabic word *algebrista*. In a mathematical sense, solving an equation (determining values of a variable that make the equation true) is like restoring, mending, or resolving it.

Hisab al-jabr w'al-muqabalah refers to the two principle operations that al-Khowarizmi used in solving an equation:

Principle 1. *al-jabr*, the transposition of terms from one side of an equation to the other.
Principle 2. *al-muqabalah*, the cancellation of equal terms appearing on opposite sides of the equation.

Let's see what these principles mean using modern symbolic algebra. Let's say that we have this equation:

$$2x + 3 = 4x^2 + 2x - 9$$

Applying *al-jabr* transforms the equation to:

$$2x + 12 = 4x^2 + 2x$$

[1] Positional notation is a system of writing numbers in which the position of a digit affects its value.
[2] Howard Eves, *An Introduction to the History of Mathematics* (New York: Holt, Rinehart and Winston, [1953, 1964, 1969] 1976), p. 193. An English translation was compiled long ago under the title, *The Algebra of Mohammed ben Musa*, ed. and trans. F. Rosen (London: Printed for the Oriental Translation Fund, 1831). al-Khowarizmi's work was very popular among medieval scholars and they slightly changed his name to "algorithm" in their representation of the subject matter.
[3] *Ibid.,* pp. 193-194.

What did we do? We transported 9 from the right hand side of the equation to the left hand side by adding 9 to both sides.

Applying *al-muqabalah* transforms the equation to:

$$12 = 4x^2$$

What did we do? The 2x on the left and right hand side of the equation "cancel" out. In other words, we subtracted 2x from both sides (or members) of the equation.

Hisab al-jabr w'al-muqabalah became known in Europe through Latin translations and was shortened to the one word, "algebra," which became synonymous with the science of equations.[4] Both Hindu and Arab algebraists were still tied to what is called "syncopated algebra" of Diophantus (known as the "father of algebra"), a 3rd century AD Greek mathematician who lived in Alexandria, Egypt. Arabic mathematicians did not use simple symbols like the symbolic algebra we used today (e.g., x, y, etc.). They used words, or at best, abbreviations of words.

The decimal or positional number system needed Simon Stevin (1548-1620) and François Vieta (1540-1603) in the sixteenth century to acquire the conceptual precision demanded by systematic scientific work.

The very soul of science consists in theoretical generalizations leading to the formulation of quantitative laws and systems of laws. In contrast, India had practicality, craftsmanship and organizational talent, but *science in the above sense did not exist in India* prior to its invasion by Alexander the Great. In the words of Tobias Dantzig:

> Algebra... enables one to transform literal expressions and thus to paraphrase any statement into a number of equivalent forms ... it is this power of transformation that *lifts algebra above the level of a convenient shorthand* ... the literal notation made it possible to pass from the individual to the collective, from the 'some' to the 'any' and the 'all' ... It is this that made possible the general theory of functions, which is the basis of all applied mathematics.[5]

Notice that Dantzig is embracing philosophy in his comments. The phrase "individual to the collection" (or, particular to the universal) reminds us of a problem that has hounded philosophers for ages, the problem of the "one and the many." Suffice it to say for now that the Biblical view of God (He is Triune, the Ultimate "One and the Many" and the world (the world is understandable because its Creator is the ultimate in rationality) enabled man to harness the incredible power of algebra.[6]

The power of algebra is in its language; it is a way in which knowledge of the patterns of creation can be expressed, developed, and used to the benefit of mankind. Algebra is the means by which the patterns of the created reality can be understood, harnessed, and applied. In the words Robin G. Collingwood, "The possibility of an applied mathematics is an expression, in terms of natural science, of the Christian belief that nature is the creation of an omnipotent God."[7]

English philosopher and scientist Roger Bacon (ca. 1214- ca. 1294), nicknamed the *Admirable Doctor* ,said in his *Opus Majus* (published in 1267), Part 4: "Mathematics is the gate and key of the sciences…. Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or things of this world."[8] What is the gate and key of mathematics? Algebra, for it prepares us not only to understand the sciences, but also such subjects as medicine, art, statistics[9], and economics. From the Biblical

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[4] *Ibid.*, p. 19.

[5] Tobias Dantzig, *Number: The Language of Science* (New York: Doubleday Anchor, [1930] 1954), p. 89.

[6] See the many historical works by the theologian/physicist Stanley L. Jaki (1924-) in this regard.

[7] Robin G. Collingwood, *An Essay on Metaphysics* (London: Oxford University Press, 1940), p. 253.

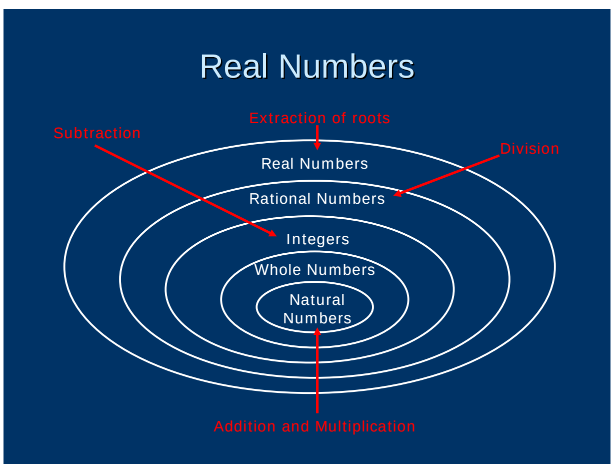
[8] Cited in Robert Edouard Moritz, *On Mathematics and Mathematicians* (New York: Dover Publications, 1958), p. 41.

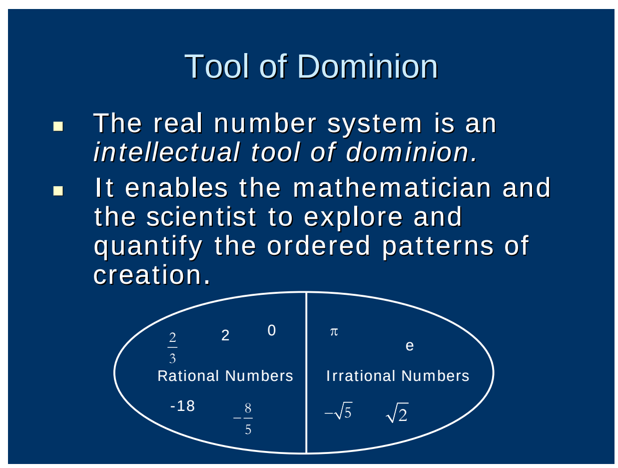
[9] These techniques are used to obtain vital knowledge about the distribution of height, weight, intelligence, mortality, income, and other facts of interest and concern to every would-be educated person. The *efficacy* of medical treatments; the *control of quality* in

Christian world view, learning algebra is important to our worship because algebra is the language of science, science is the study of what God has made, and what God has made is an unveiling of His glory.

With this history, etymology, and world view analysis behind us, let's now explore the inner workings of algebra. According to Warren Esty, Professor of mathematics at Montana State University, "In arithmetic, the emphasis is on numbers. In algebra and calculus the emphasis is *beyond* numbers. Algebra is about operations and order."[10]

The numbers assumed to exist in every algebraic equation are the *real numbers.*[11] You can build the set of real numbers piece by piece. The first piece is the set of positive integers (or natural numbers): 1, 2, 3, 4, … . Adding 0 extends this set to set of whole numbers: 0, 1, 2, 3, 4, … . The operations of addition and multiplication are *closed* in both the set of natural and the set of whole numbers. Closure means that when I add or multiply any two whole numbers or any two natural numbers, the sum or product will always be a whole number or a natural number. We extend the whole numbers to get the set of integers: …, -3, -2, -1, 0, 1, 2, 3, … . The set of integers is closed under the operation of addition, multiplication, and subtraction. Extending further, we get the set of rational numbers (fractions or quotients of integers as long as the denominator or divisor is not equal to 0). The set of rational numbers is closed under the operation of addition, multiplication, subtraction, and division. A real number that is *not* a rational number is said to be *irrational.* For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and π (the ratio of the circumference to the diameter of a circle) are all examples of irrational numbers.[12] The set of real numbers is closed under the operation of addition, multiplication, subtraction, division, and extraction of roots. The real number system is an *intellectual tool of dominion.* These numbers enable a scientist to explore and quantify the ordered patterns of creation.





Algebra is foremost a language and a language is governed by the proper use of syntax along with a systemized set of notations. Algebra, being a unique type of language replete with symbols and methods, is all about *operations and order.* Once we park this concept in our "little grey cells," then there is hope for us in understanding and mastering it.

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production; and the *prediction of future* prices of commodities, population growth, and genetic traits such as susceptibility to diseases are achieved with the same tools. The reliability of conclusions reached on the basis of statistics and probability should also be taught. The well-known quip, "*There are lies, damned lies, and statistics*," should certainly be taken seriously in the study of statistics, because so many of the conclusions that are hurled at us are not supported by the data.

[10] Warren Esty, *Pre-Calculus, Third Edition* (Bozeman: Montana State University, 2006), p. 1. I am indebted to the teaching emphasis of Professor Esty for many of the principles taught in the rest of this essay.

[11] The adjective "real" was originally used to differentiate these numbers from numbers like $\sqrt{-1}$, which were at one time thought to be "unreal."

[12] Proving that these numbers are irrational requires some ingenuity of logic.

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Let's look at a few examples to understand the importance of "operations and order." Let's say that we want to solve this equation for x:

Equation 1. $3(x - 8) = 6$

Rhetorically, this equation is a sentence. It says, "3 times the difference of a certain number and 8 is 6." The expressions on the left side and right side of the equation are both called *algebraic expressions*. An equation is setting two algebraic expressions equal to each other. Every equation consists of the equal sign (=) that divides it into two algebraic expressions, its left member and its right member.

Solving equations involving an unknown (as in our example) means we must find the value of the unknown that makes the equation true; i.e., the left member equals the right member. In other words, what is the value of x such that $3(x - 8) = 6$?

Let's go ahead and solve this equation algebraically. To begin with, let's divide both members by 3. What happens when we do so?

$$\frac{3(x-8)}{3} = \frac{6}{3} \Leftrightarrow x - 8 = 2$$

By this process of dividing both members by the same number, we have generated an *equivalent equation*. In other words, $x - 8 = 2$ is saying the same thing as $3(x - 8) = 6$. What we have done is a revelation of a basic *operation* of algebra. You can add, subtract, multiply, or divide (except by 0) both members of an equation by the same number *or* algebraic expression and your result will be an equivalent equation.

> Algebra is all about *operations and order*.

We now have the equation $x - 8 = 2$. If we add 8 to both members, we get:

$x = 10$

Halt! We have solved the equation! We have found the value of 10 that makes $3(x - 8) = 6$. To check our solution, we replace x with 10 and see what happens: $3(10 - 8) = 3(2) = 6$. Check! The left member equals the right member.

Let's solve another equation for x:

Equation 2. $6(x - 34) = 96$

Although the Equation 2 is different from Equation 1, *the order of operations is the same*. Rhetorically, it says, "6 times the difference of a certain number and 34 is 96." We solve it in the same fashion:

$$\frac{6(x-34)}{6} = \frac{96}{6} \Leftrightarrow x - 34 = 16 \text{ (divide both members by 6)}$$

$x - 34 = 16 \Leftrightarrow x = 50$ (add 34 to both members)

I repeat, although the numbers have changed, the operations are the same. To solve Equation 1, we first *divide* both members by 3 and then *add* 8 to both members. To solve Equation 2, we first *divide* both members by 6 and then *add* 34 to both members. The solution to Equation 1 depends upon the *sequence of operations* expressed by $3(x - 8)$ and the solution to Equation 2 depends upon the *sequence of operations* expressed by $6(x - 34)$. Hence, the key to solving equations is knowledge of the *sequence or order of operations*. Hence, algebra is all about *operations and order*.

Let's look at two more equations together that we desire to solve.

Equation 3. $3(x - 8) = 6$

Equation 4. $3(x - 8) + x = 6$

Equation 3 is the same as Equation 1, but Equation 4 is different. What is different about Equation 4? An extra *operation*, + x. We need to work with the left member to simplify it because dividing both members by 3 will not help us the way it helps us to solve Equation 3.

First, we change the *operations and order* by noting that $3(x - 8) = 3x - 24$.

Replacing $3(x - 8)$ with $3x - 24$, we get:

$3x - 24 + x = 6$

Second, we change the *operations and order* again by "combining like terms."[13] We note that $3x + x = 4x$.[14]

Replacing $3x + x$ with $4x$, we get:

$4x - 24 = 6$

Third, we add 24 to both members. We get:

$4x = 30$

Fourth and finally, we divide both members by 4. We get:

$$x = \frac{30}{4} = \frac{15}{2} = 7.5$$

To solve Equation 4, we had to expand $3(x - 8)$ while in Equation 1, we did not. The requirement to expand $3(x - 8)$ was necessitated by the extra *operation*, + x.

Solving Equation 4 used a process called the distributive property. $3(x - 8) = 3x - 24$. Look at the left member. It means "3 times the difference between a certain number and 8." The right member is another way, an equivalent expression, of saying the same thing. We distribute the 3 to x (by multiplying to get 3x) and we distribute the 3 to -8 (by multiplying to get -24). The distributive property is one of the "bread and butter" *identities* of algebra. An identity is an equation that is true for every value of the variables. In general, three numbers are involved. We let the letters a, b, and c stand for these numbers. We assume that these numbers are real numbers. Hence, the *Distributive Property* is:

$$a(b + c) = ab + bc,\ (a + b)c = ac + bc$$
or
$$a(b - c) = ab - bc,\ (a - b)c = ac - bc$$

The first identity is technically called the Distributive Property of Multiplication over Addition. The second identity is called the Distributive Property of Multiplication over Subtraction. There are related identities for distributing division over addition and distributing division over subtraction. They are:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \Leftrightarrow \frac{1}{c}(a+b) = \frac{a}{c} + \frac{b}{c}$$
or
$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c} \Leftrightarrow \frac{1}{c}(a-b) = \frac{a}{c} - \frac{b}{c}$$

In the words of Professor Esty, "Identities express alternative sequences of operations."[15] This means

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[13] In algebra, a term is of the form ax^n where x^n is called the "literal part" and a is called the "coefficient." "Combining like terms" means to join two or more terms that have the same literal parts by adding or subtracting their respective coefficients. Note, in our example $3x = 3x^1$ and $x = 1x^1$.

[14] To do this, we can rearrange the terms of the left member of the equation; i.e., $3x - 24 + x = 3x + x - 24$.

that you can *substitute* either order (either member of the equation) for the other. As we have noted, we used the Distributive Property to solve Equation 4. We also used it a second time. Do you know where? $3x + x = x(3 + 1) = 4x$.[16]

$3(x – 8) = 3x – 24$ is an identity. It represents the equivalency of two algebraic expressions. Does this equivalency say anything about x, the unknown? No. *It says nothing about x*. It is true no matter what number you replace x by. This identity tells us *nothing* about x, but it tells us *something* about *operations and order*. The operations in the left member, in the order in which they are processed, are "subtract 8," then "multiply by 3." The operations in the right member, in the order in which they are processed, are "multiply by 3," then "subtract 24." Both operations, the left member and the right member, will generate the same result for any x. This is why $3(x – 8) = 3x – 24$ is an identity.

In contrast, Equation 3, $3(x – 8) = 6$, tells us something about the number x. It tells us that $x = 10$. To solve equations like this, you must see and employ a *sequence of operations*. So, algebra is not just about numbers. Algebra concerns numbers (they are all over the territory), but algebra is primarily about something more; it is about operations and the order in which they are performed. Although operations and order help us solve equations (give us a numerical result), operations and order are much more sophisticated than numbers.

> Algebraic identities are all about *operations and order*.

Besides the Distributive Property, there are many other important identities used constantly in algebraic operations. We have used a few of these identities already. Here are most of them, without comment[17]:

- $0 + a = a + 0 = a$
- $a + (-a) = 0$ and $-a + a = 0$
- $a + b = b + a$
- $(a + b) + c = a + (b + c)$
- $a = -(-a)$
- $-(a + b) = -a – b$
- $ab = ba$
- $(ab)c = a(bc)$
- $1a = a$ and $0a = 0$
- $a\left(\frac{1}{a}\right) = \frac{a}{a} = 1$
- $(-1)a = -a$
- $-(ab) = (-a)b = a(-b)$
- $(-a)(-b) = ab$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a – b)^2 = a^2 – 2ab + b^2$
- $(a + b)(a – b) = a^2 – b^2$

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[15] Esty, p. 2.

[16] Recall that in multiplication, it does not matter in which order you arrange the factors; i.e., ab = ba. In our example, $3x + x = x(3 + 1) = x4 = 4x$.

[17] Knowledge of these identities should be mastered in an elementary algebra course.

- $a^1 = a$
- $a^m a^n = a^{m+n}$
- $a^0 = 1$
- $(a^m)^n = a^{mn}$

Functions are very important in mathematics. A function is basically a rule or an algebraic operation; i.e., an algebraic procedure applied to every input of the function that generates a unique output. Let's let our rule be "square all inputs." We let x represent any input, we denote the rule applied to the input as f(x); we say "f of x". Hence, in algebraic symbols, the squaring function is represented as $f(x) = x^2$. We let our input be $\frac{a}{3}$. We get: $f(x) = f\left(\frac{a}{3}\right) = \left(\frac{a}{3}\right)^2 = \frac{a^2}{9}$. We note that $\left(\frac{a}{3}\right)^2 = \frac{a^2}{9}$ is true for all a. This is also an identity. The left member tells us that we divide by 3 and then square. The right member tells us that we square and then divide by 9. Both operations result in the same answer. Note again that this identity tells us nothing about *a*. *The identity informs us of the order of operations.*

> **Functions are all about *operations and order*.**

I hope that you can understand better how algebra is a language; it is a syntax involving operations and the order in which they are processed. Algebraic expressions are the "nouns and pronouns" of this language. The equal sign is equivalent to the verb "to be." Algebraic expressions can be numbers (e.g., $2(4^2)$, $\frac{\pi}{4}$, 88, etc.) or the can include variables (e.g., x^2, $\frac{3x}{2}$, (5 + 8)x, tan x,[18] 3xy, 15z – 8y, etc.).

Algebraic expressions with variables represent two different ideas. First, the variable can represent a number. Every elementary course in algebra makes sure that you understand this. The phrase, "let x represent a number," is used repeatedly. The second idea is that the variable can represent an *order of operations*, an order that does *not* depend upon the value of the variable. For example, the algebraic expression (5 + 8)x tells us something about the sequence in which the operations of addition and multiplication are to be performed. We first *add* 5 to 8. Then, we take that sum, 13, and *multiply* by x.

To help us differentiate between an equation using a variable as a number and an equation using a variable as an identity, let's look at a couple of examples.

Equation 5. $(x + 3)^2 = 4$
Equation 6. $(x + 3)^2 = x^2 + 6x + 9$

In Equation 5, x represents a number. This equation can be solved. Equation 6 is an identity. It involves the operations of addition and squaring. It says "add 3 to a given number and then square the result" is equivalent to "square the given number, add 6 times the given number, and then add 9."

Consider these two equations.

Equation 7. $x^2 – 9 = (x + 3)(x – 3)$
Equation 8. $x^2 – 9 = 27$

Equation 7 is an identity that is about giving alternative operations in different orders while Equation 8 can be solved; i.e., x represents a number.

What have we learned so far?

[18] tan x is a representation of a trigonometric or circular function.

1. Equations can be solved applying operations in a certain order.
2. Identities reveal different ways on applying operations and their order.
3. Functions are rules that reveal something about operations and their order.
4. Identities and equations look *similar* but they are *different* in meaning.

Scientific laws are represented using algebraic formulas. For example, since the ratio of the circumference, C, to the diameter, d, of a circle is π, then we can write a formula for C; i.e., $C = \pi d$. Similarly, the ratio of the area, A, of a circle to the square of its radius, r, is π. Hence, we can write a formula for A; i.e., $A = \pi r^2$. These formulas are not primarily about numbers (although they concern numbers); they are about operations and order.

Formulas are all about *operations and order*.

Other well-used and elementary formulas:

A, the area of a square of side *s*: $A = s^2$

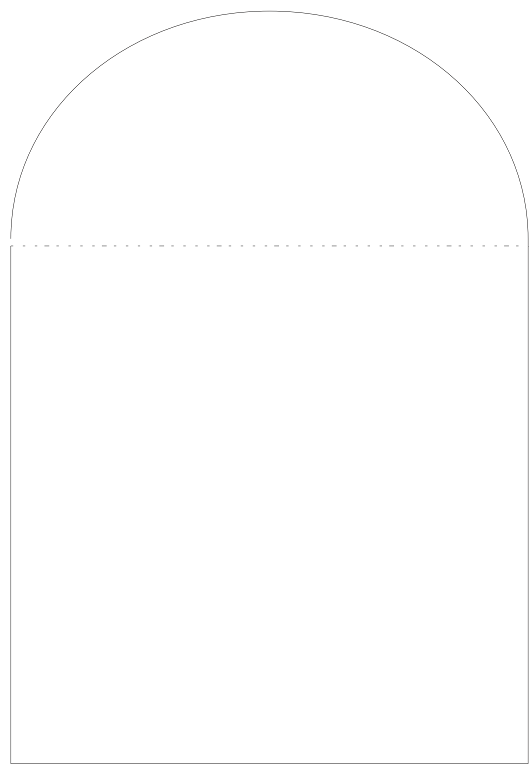
A, the area of a triangle of base *b* and height *h*: $A = \frac{1}{2}bh$

A, the area of a rectangle of length *l* and width *w*: $A = lw$

P, the perimeter of a square of side *s*: $P = 4s$

P, the perimeter of a triangle with sides of length *a*, *b*, and *c*: $P = a + b + c$

P, the perimeter of a rectangle of length *l* and width *w*: $P = 2w + 2l$



If we know that the lengths of the three sides of a triangle are 3 cm, 4 cm, and 5 cm, then we can determine its perimeter using arithmetic; i.e., 3 cm + 4 cm + 5 cm = 12 cm. No algebra is needed to do this.

However, let's say that Farmer Jones ploughs a field with a shape of a square with a cap that is a semicircle. Let's say that the perimeter of this field is 600 meters. We want to find the length of the side of the square. How would we proceed logically?

We can use our formulas to help us construct a new formula to answer this question. We know that, for a square, $P = 4s$ and for the circumference of a semicircle, $C = \frac{\pi d}{2}$. We also know that $d = s$ (the diameter of the semicircle is one of the sides of the square). Since we want to find the side of the square, we let x represent this unknown. Hence, by substitution, $P = 4x$ and $C = \frac{\pi x}{2}$. We know that the perimeter of the field is 600 meters. 600 meters is equal to the length of *three* sides of the square plus the circumference of the semicircle. Hence, we can write an equation:

$$3x + \frac{\pi x}{2} = 600$$

This is an equation that can be solved. It is *not* an identity. We shall use operations in a certain order to solve this equation. First, inspect the left member. Can it be simplified? We see that x is common to both terms. We can invoke the Distributive principle:

$$3x + \frac{\pi x}{2} = x(3 + \frac{\pi}{2})$$

Our equation is now:

$$x(3 + \frac{\pi}{2}) = 600$$

We see that $(3 + \frac{\pi}{2})$ is just a number. The left member states that a certain number x is multiplied by $(3 + \frac{\pi}{2})$. The equation states that x multiplied by the number $(3 + \frac{\pi}{2})$ is 600. If we divide the left member and the right member by $(3 + \frac{\pi}{2})$, we get (since $\frac{3+\frac{\pi}{2}}{3+\frac{\pi}{2}} = 1$ and $1x = x$):

$$x = \frac{600}{3+\frac{\pi}{2}}$$

We have our formula! Note that the logical process employed (a sequence of operations) to get this formula is *independent* of the perimeter of the field. We could let P = the perimeter of the field and our formula would be:

$$x = \frac{P}{3+\frac{\pi}{2}}$$

Solving this word problem concerned numbers but was primarily about operations and order. Our solution is:

$$x = \frac{600}{3+\frac{\pi}{2}} \approx 131.27 \text{ meters}$$

Word problems are all about *operations and order*.

Finding the perimeter of a triangle knowing its three sides is easy and direct (we just add). In contrast, solving our field problem was not easy and not direct. The problem suggested to us that we need to use some given formulas. However, we could not answer the problem *directly* from these formulas. We had to use our "little grey cells" and logically generate a new formula employing a sequence of operations. What we did to solve this problem, employing a sequence of operations, is what algebra is all about.

Let's look at one more example, a physics problem, to illustrate why algebra is all about operations and order.

Given a moving object, we define its velocity, *v*, as its change in position, *p*, over a period of time, *t* $\left(v = \frac{\Delta p}{t}\right)$[19], and its acceleration, *a*, as its change of velocity over a period of time $\left(a = \frac{\Delta v}{t}\right)$. The amount of acceleration an object undergoes depends upon the strength of the force applied. The stronger the force, the *more* the object accelerates. Hence, acceleration of an object is directly proportional to force, *f*. In symbols (where $\propto$ means "is proportional to"):

$$a \propto f$$

The acceleration of an object also depends upon its mass.[20] The more massive the object, the *less* the object accelerates. Hence, acceleration of an object is indirectly proportional to the object's mass, *m*. In symbols:

$$a \propto \frac{1}{m}$$

Combining both proportions, we get:

$$a \propto \frac{f}{m}$$

As an equation, $a = \frac{kf}{m}$ where k = constant of proportionality. Based upon units chosen for force and mass, k can be set equal to 1. Hence, $a = \frac{f}{m}$ or, multiplying both members by m (*an operation*), f = ma (this is Isaac Newton's second law of motion). f = ma represents the reality of God's covenantal order in everyday familiarity. We experience this physical reality (minus friction and air resistance) because throwing (or, exerting a force) a baseball is much easier than throwing a bowling ball.

When we release an object of certain mass from a tall building, it falls. In physics, it undergoes a force, or a pull, called gravity. Gravity is a physical phenomenon that is universal in the created order. Gravity, reflecting God's creational and covenantal order, generates a force between every pair of objects in the universe. The force of gravity between any two objects depends directly upon the masses of those objects and indirectly upon the distance, indeed the square of the distance, between these two objects. A baseball dropped from a tall building has mass. It falls down because the earth, being of greater mass, exerts a force, called gravity, and pulls the baseball straight down in a direction towards the center of the earth.

Quantitative Physics is all about *operations and order*.

An object's weight is the measure of how hard gravity pulls on it. Mass is different from weight because it is the measure of how difficult it is for an object to accelerate. Because of this, an object that is heavy (in weight) is also difficult to accelerate (or move).

An object's weight, *w*, is proportional to the "local strength of gravity" or the acceleration due to gravity, *g* (a spring scale measures this force). An object's weight is also proportional to its mass. In symbols, we

[19] Δ is the Greek letter delta and, in physics and mathematics, it is symbol that means "change in."
[20] Mass is the measure of an object's inertia or resistance to changes in velocity.

have:

$$w \propto g \text{ and } w \propto m$$

Combining both symbols, we get:

$$w \propto mg$$

As an equation, we get w = kmg where, again, k is the constant of proportionality. We can choose units so that k = 1. Hence, we get;

$$w = mg$$

What do the *operations* of this equation mean? If means that you can lose weight either by reducing your mass or by going someplace, like the moon, where the gravity is weaker.

What do we mean by acceleration due to gravity? When we drop a ball, it accelerates downward according to Newton's second law, $a = \frac{f}{m}$. Since the ball is falling, the only force on it is its weight. That weight is equal to its mass times its acceleration due to gravity, or w = mg. This analysis requires us, algebraically, to equate f with w; i.e., f = w. Now watch as algebraic order and operations reveal to us a connection:

$a = \frac{f}{m}$

Since, f = w, then, by substitution, $a = \frac{w}{m}$

Since w = mg, then, by substitution, $a = \frac{mg}{m}$

Since $\frac{m}{m} = 1$, then, by substitution, a = g.

a = g means that the falling ball's acceleration is equal to the acceleration due to gravity. Hence, acceleration due to gravity is really acceleration; it is the acceleration of a freely falling object. The operations and order of algebra *require* us to come to this conclusion.

In conclusion, operations and order are what algebraic is all about. The syntax of algebra is designed to express operations and order. Since God's creational covenants are orderly (cf. Genesis 8:21-22; 9:11), then algebra, in a derivative sense, reflects on that same order (as we have seen with this physics example). Algebra and the subsequent mathematics built upon it (e.g., calculus) are all about operations and order. To solve any equation, you must be able to see and implement operations in a certain order. Identities and functions are all about operations and order. Identities reveal a different sequence of operations. Functions are expressed in terms of a variable x but they are not *about* x. Formulas and word problems use symbols and their manipulation invokes proper ordering of operations. Problems in algebra and science (which uses algebra) are usually indirect. In formulas and in scientific analysis, you manipulate operations that can be applied to specific numbers but when you *do* algebra in these contexts, you must always consider "operations and order."

God's creational covenants are all about *operations and order*.