The History and Mathematics of Conversions

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THE METRIC SYSTEM I

If you live in the United States of America, you have to work with two systems of measure. These two systems are called (1) British Imperial system of measure and (2) Metric system of measure. In the British system, with its long and storied history, there are many sub-systems where different bases are used. The pint-gallon system is base 8, the inches-foot system is base 12, the yard-foot system is base 3, the week-day system is base 7, the month-year system is base 12, the yard-mile system is base 1760, and the foot-mile system is base 5280.

In contrast, the Metric system of measurement is, like most national currencies, *decimalized* (base 10). As we have already noted, it was developed in France in the late 18th century.[1] Since the 1960s the International System of Units (SI) (*Système International d'Unités* in French, hence "SI") has been the internationally recognized standard metric system. Metric units are widely used around the world for personal, commercial and scientific purposes. Of all the nations of the world (Date: early 21st century), only Liberia, Myanmar and the United States have not yet *officially* adopted the Metric system.

Nations in red have not officially adopted the Metric System (Public Domain)

Metric units consist of a *standard* set of prefixes in *multiples of 10* that may be used to derive larger and smaller units. Working with these units is as easy as multiplying or dividing by 10 (or powers of 10).

Before we inspect the nature of the metric system, let's review the British Imperial system as a basis for comparison. Let's first establish some definitions. A **measure** is a *standard unit* established either by law or custom, by which *quantity*, such as extension, dimensions, capacity, amount, or value, is measured or estimated. Measures can be classified in six ways:

1. Extension.
2. Capacity.
3. Weight.
4. Time.
5. Angles.
6. Value (e.g., money).

Let's briefly consider each classification. **Extension** is that which has one or more dimensions (length, area, and volume have one, two, and three dimensions, respectively). Here are some typical linear or

During the reign of King Henry I (1068-1135) of England, he decreed that the distance from the end of his nose to the end of this thumb was to be the lawful yard.

Jules Verne (Public Do-

In Germany in the 16th century an attempt was made to standardize the rod. To secure a standard it was agreed that the length of a rod was to be equal to the length of the left feet of *sixteen* men when they were standing heel to toe.

[1] Thomas Jefferson (1743-1826), third president of the United States, introduced the decimal money system to America because his "French connection."

one-dimension measurement (we have already encountered some of them and others are of genuine historical interest).

3 barley-corns[2]	=	1 inch (in.)
9 inches (in.)	=	1 span[3]
12 inches[4]	=	1 foot (ft.)
21.888 inches	=	1 sacred cubit[5]
8 spans	=	1 fathom[6]
3 feet	=	1 yard[7] (yd).
3 feet	=	1 pace
6 feet	=	1 fathom
5½ yards	=	1 rod[8]
16½ feet	=	1 rod
660 feet	=	1 furlong[9] (fur)
40 rods	=	1 furlong
4 rods	=	1 chain[10]
22 yards	=	1 chain
80 chains	=	1 mile (mi)[11]
320 rods	=	1 mile
5280 feet	=	1 mile
$1.152\frac{2}{3}$ miles	=	1 nautical mile or knot[12]
3 nautical miles	=	1 league[13]
60 nautical miles	=	1 degree[14]
360 degrees	=	circumference of the earth at the equator

[2] The length of three barley corns placed end to end was established as an inch in 14th century England. In the next lesson, we shall see the connection between a barley-corn and weight.
[3] The *span* was originally defined as the distance between the tip of the thumb and the tip of the little finger when a hand is fully extended.
[4] The word *inch* comes from the Latin *uncial,* meaning "one twelfth."
[5] *Cubit* is a unit of measure used in the Old Testament Scriptures. Originally, the cubit was a unit of measurement equal to the length of the forearm from the point of the elbow to the end of the middle finger. The ancient Egyptian cubit was about 20.7 inches and the ancient Roman cubit was about 17.5 inches. The Hebrew measurement probably fell between these two lengths.
[6] A *fathom* is used to measure depths at sea. It originally meant the measure of the span (distance) between two outstretched arms. Historically, a *cable* is 120 fathoms.
[7] A *yard*, in early medieval times meant the distance of a "straight twig." As we have noted, legend states that Henry I of England defined it as the distance between the tip of his thumb and the tip of his nose with his arm outstretched; another theory states that it derives fro the girth of a person's waist.
[8] A *rod* was a length of a standard perch or pole.
[9] *Furlong* is derived from "furrow length," the distance plowed in one acre, stretched out in a straight line. 660 feet is one-eighth of a mile, meaning that there are 8 furlongs in one mile. The furlong's use is now largely restricted to British horse racing.
[10] Commonly used by surveyors. A *chain* is a distance-measuring device consisting of a chain of 100 links of equal length.
[11] This *mile* is officially called a *statute mile* and is used for land measurement. Its probable origin is from the Latin *millia passuum* , "1000 paces," with 1 pace equal to about 5 feet.
[12] Used to measure distances at sea. *Knot* is used to measure the speed of ships. *1 knot* was originally defined as "1 unit of 47 feet, 3 inches on a log line, marked off by knots."
[13] Recall the science fiction book by Jules Verne (1828-1905) entitled *Twenty Thousand Leagues under the Sea* (first published in 1870). A league was originally defined as the distance a man or a horse could walk in one hour.
[14] Of latitude or longitude on the equator.

Surface or **square measure** is used in computing areas or surfaces. Hence, a surface has two dimensions (length and width). As we have already noted, the **area** of a surface is calculated as the product of the numbers that represent these two dimensions. Also, recall that a **square** is a plane figure (two dimensions) bounded by four equal sides having four right angles. Here are some typical surface measures.

144 square inches (in^2)	=	1 square foot (ft^2)
9 square feet	=	1 square yard (yd^2)
30.25 square yards	=	1 square rod (rd^2)
160 square rods	=	1 acre[15]
1 square chain	=	1 acre
640 acres	=	1 square mile (mi^2) or 1 section of land
36 square miles	=	1 township[16]

Cubic or **solid measure** is used in computing the contents or volumes of solids. A **solid** has three dimensions (length, width, and height). As we have already noted, the **volume** of an object is calculated as the product of the numbers that represent these three dimensions. Also, recall that a **cube** is an object bounded by six equal squares called **faces**. Here are some typical solid measures.

16 cubic feet (ft^3)	=	1 cord foot (cd ft)[17]
8 cord feet	=	1 cord (cd)
128 cubic feet	=	1 cord (cd)

In 1751, Jedediah Buxton of Elmton, Chesterfield, Derbyshire showed that 1 in^3 was the same as:

- 200 barleycorns
- 300 wheatcorns
- 512 ryecorns
- 180 oats
- 40 peas
- 25 beans
- 80 vetches (a bean-like seed)
- 100 lentils
- 2304 one inch long hairs

Now, let's consider the metric system as it applies to extension. The **meter** (m) is the *base* of the metric measure of extension and it is approximately equal to 3.3 feet. The meter is used from measuring short distances. Starting with the meter, units of greater dimension are powers of ten and have prefixes from the *Greek* language that indicate those powers.

10^1 = 10 meters	=	1 decameter or dekameter (dam or dkm)	1 m ≈ height of door knob from the floor
10^2 = 100 meters	=	1 hectometer (hm)	
10^3 = 1000 meters	=	1 kilometer (km)	1 km ≈ length of about five city blocks
10^4 = 10,000 meters	=	1 myriameter (mym)	
10^6 = 1,000,000 meters	=	1 megameter (Mm)	
10^9 = 1,000,000,000 meters	=	1 gigameter (Gm)	

The most common units used are the meter and the kilometer (1 km ≈ 0.62137 miles) although you will see the prefixes mega- and giga- used with reference to computer data and memory storage. Kilometers are common used for measuring long distances. Decameters, hectometers and myriameters are very rarely used.

[15] The word *acre* comes from medieval times. It originally meant the amount of land one man could plough with one ox in one day. Later, a more measurable standard was set: one acre = a strip of land 4 rods wide and one furrow long; i.e., 4 rods by 1 furlong, 4 rods by 40 rods, or 160 square rods.

[16] A *township* (meaning "community") is a unit of local government or a subdivision of a county as defined in most Midwestern and Northeastern states in America and in most Canadian provinces.

[17] A *cord* is a wood measure; it is still a common unit of measure in areas where firewood is sold and used.

THE METRIC SYSTEM I

Units smaller than a meter (by factors of 10) have prefixes from the *Latin* language (note especially the use of negative exponents, $\frac{1}{10} = 10^{-1}$, as an introduction to our studies in Tour 10).

$10^{-1} = \frac{1}{10} = 0.1$ meters	=	1 decimeter (dm)	1 dm ≈ diameter of a large orange
$10^{-2} = \frac{1}{100} = 0.01$ meters	=	1 centimeter (cm)	1 cm ≈ width of a shirt button; 1 ft ≈ 30 cm
$10^{-3} = \frac{1}{1000} = 0.001$ meters	=	1 millimeter (mm)	1 mm ≈ thickness of a small coin
$10^{-6} = \frac{1}{1,000,000} = 0.000001$ meters	=	1 micrometer (μm)[18]	1 μm ≈ diameter of one bacterial cell
$10^{-9} = \frac{1}{1,000,000,000} = 0.000000001$ meters	=	1 nanometer (nm)	1 nm ≈ thickness of one RNA molecule

Units smaller than a meter (by factors of 10 have prefixes from the *Latin* language. The most common units used are the centimeter (1 cm ≈ 0.3937 inches) and the millimeter (used to measure very small distances) although you will see the prefixes micro- and nano- used with reference to atomic measurements. Decimeters are very rarely used.

The wonder of the metric system is that these same prefixes are used for measurements of capacity and weight. Hence, if you know the meanings (in terms of powers of ten) of these prefixes in one context, you can immediately apply the same meaning to other others.

Surface measurements use the same prefixes, as follows:

100 square millimeters (mm^2)	=	1 square centimeter (cm^2)
100 square centimeters	=	1 square decimeter (dm^2)
100 square decimeters	=	1 square meter (m^2)
100 square meters	=	1 square decameter (dam^2)
100 square decameters	=	1 square hectometer (Hm^2) = 1 hectare (ha) ≈ 2.471 acres
100 square hectometers	=	1 square kilometer (km^2)

A square meter is the unit for measuring ordinary surfaces of small extent (e.g., floors, ceilings, etc.). The hectare is used to measure larger land measures.

Volume measures are noted as follows:

1000 cubic millimeters (mm^3)	=	1 cubic centimeter (cm^3 or cc)	1 cm^3 ≈ cube of sugar
1000 cubic centimeters	=	1 cubic decimeter = 1 liter (L)	
1000 liters	=	1 cubic meter (m^3) = 35.3165 ft^3 = 0.2759 cord	

A liter is also a unit of volume (or capacity) for both dry and liquid measures. **Capacity** signifies room or space. The common connection between capacity and volume is the fact that 1000 cm^3 = 1 liter. 1 liter is equal to 1.0567 quarts (liquid measure; e.g., measure of gasoline) and 1 liter is equal to 0.908 quarts (dry measure; e.g., measure of a container of flour). Liters can be used as a base unit with the other prefixes as follows:

[18] μ is the Greek letter "mu."

1000 microliters (μL)	=	1 milliliter (mL)	1 μL ≈ crystal of table salt
10 milliliters	=	1 centiliter (cL)	1 mL ≈ 20 drops of water
10 centiliters	=	1 deciliter (dL)	
10 deciliters	=	1 liter	1 L ≈ quart of milk
10 liters	=	1 decaliter (daL)	
10 decaliters	=	1 hectoliter (hL)	
10 hectoliters	=	1 kiloliter (kL)	

Concerning liquid and dry measures, here is a good "kitchen" equivalency:

Kitchen *liquid* measures:

4 quarts (qt)	=	1 gallon (gal)
2 pints (pt)	=	1 quart
2 cups	=	1 pint
8 fluid ounces[19] (fl oz)	=	1 cup
1 cup	≈	250 mL
1 pt	≈	0.5 L

Hence, 1 gal = 4 qt = 8 pt = 16 cups = 128 fl oz ≈ 3.79 L.

Kitchen *dry* measures:

16 tablespoons (Tbs)	=	1 cup
3 teaspoons (tsp)	=	1 tablespoon (Tbs)
1 teaspoon	≈	5 mL
1 tablespoon	≈	15 mL

Hence, 1 cup = 16 tablespoons = 48 teaspoons = 240 mL.

Source: iStockPhoto

The requirement to calculate conversions between units is a common theme in measurement. Follow this logic when making conversions between metric units.

Step 1. Ask yourself, "Which unit is the larger unit?"

Step 2. If you are converting from a *larger unit to a smaller unit*, then *multiply* by the conversion relationship (also called the **unit multiplier**). Why? You need *more* of the smaller unit than of the larger unit to make up the given quantity. As a memory aid, picture the large unit as you are standing with both hands raised at a 45° angle to the line vertical to the top of your head. To convert from larger to smaller, you bring your arms together and cross them; i.e., × for multiplication. Another way to say this is if you are converting *down* the scale to a *smaller* unit of measure, the number of units must get *larger*.

Step 3. If you are converting from a *smaller unit to a larger unit*, then *divide* by the unit multiplier. Why? It takes *fewer* of the larger unit to make up the given


Source: iStockPhoto

[19] *Ounce* comes from the French *onze* meaning "eleven."

volume or quantity. Another way to say this is if you are converting *up* the scale to a *larger* unit of measure, the number of units gets *smaller*.

Here are some examples:

Suppose we need to convert 10 km to m. Which is larger? km. You are converting from larger to smaller (×, your crossed arms), therefore *multiply*. The unit multiplier is 1000 since 1 km = 1000 m.

$10\text{km} = 10\text{km} \times \frac{1000\text{m}}{\text{km}} = \frac{10\,\cancel{\text{km}}}{1} \times \frac{1000\text{m}}{\cancel{\text{km}}} = 10,000\text{m}$. Note especially how the units $\frac{\cancel{\text{km}}}{\cancel{\text{km}}} = 1$ and therefore cancel out. Note also that we are multiplying by 1,00 (a power of 10). By our rules of multiplying by powers of ten, we shift the decimal point in 10 three places to the right to get our answer: 10,000 m.

Now let's convert from 1600 mm to km. Which is larger? km. You are converting from smaller to larger, therefore *divide*. The unit multiplier is 1,000,000 because 1 km = 1,000,000 mm.

$1600\text{mm} = \frac{1600\text{mm}}{1} \div \frac{1,000,000\text{mm}}{\text{km}} = \frac{1600\,\cancel{\text{mm}}}{1} \times \frac{\text{km}}{1,000,000\,\cancel{\text{mm}}}$. Note again how that *mm* units cancel: $\frac{\cancel{\text{mm}}}{\cancel{\text{mm}}} = 1$. We are dividing 1600 by 1,000,000 (a power of 10). By our rules of division by powers of ten, we shift the decimal point in 1600 *six* places to the *left* to get our answer: 0.0016 km.

In summary, converting from one unit to another unit in the metric system is just a matter of shifting decimal places to the *right* (when *multiplying* by the unit multiplier) or to the *left* (when *dividing* by the unit multiplier). In converting, remember to answer two key questions:

1. Which unit is larger? This determines *whether* you multiply or divide.
2. What is the unit multiplier? This determines *what* you multiply or divide by.

For those who like a more detailed algorithmic approach to unit conversions, here is an eight-step approach.

We want to convert 80 kilometers to meters.

Step 1. If the unit of measurement that you are converting is a whole number, put it over 1. If it is a fraction, first convert it to a decimal and then put it over 1.

$$\frac{80\text{km}}{1}$$

Step 2. Place a multiplication sign next to the right.

$$\frac{80\text{km}}{1} \times$$

Step 3. Draw a fraction bar to the right.

$$\frac{80\text{km}}{1} \times -$$

Step 4. Put in the units of measurement. The unit of measurement to be removed (km) is written in the denominator. The unit of measurement to be converted (cm) is written in the numerator.

$$\frac{80\text{km}}{1} \times \frac{\text{m}}{\text{km}}$$

Step 5. Enter the unit multiplier (1 km = 1000 m) matching the units appropriately.

$$\frac{80\text{km}}{1} \times \frac{1000\text{m}}{1\text{km}}$$

Step 6. Multiply disregarding the units.

$$\frac{80\text{km}}{1}\times\frac{1000\text{m}}{1\text{km}}=\frac{80,000}{1}$$

Step 7. Reduce or simplify the fraction if necessary.

$$\frac{80\text{km}}{1}\times\frac{1000\text{m}}{1\text{km}}=\frac{80,000}{1}=80,000$$

Step 8. Cancel like units of measurement and carry over the remaining unit of measurement to the answer.

$$\frac{80\,\cancel{\text{km}}}{1}\times\frac{1000\text{m}}{1\,\cancel{\text{km}}}=\frac{80,000}{1}=80,000\text{m}$$

You are done!

Let's try one more. Convert 78 centimeters to meters.

Step 1. $\frac{78\text{cm}}{1}$

Step 2. $\frac{78\text{cm}}{1}\times$

Step 3. $\frac{78\text{cm}}{1}\times-$

Step 4. $\frac{78\text{cm}}{1}\times\frac{\text{m}}{\text{cm}}$

Step 5. Since 1 m = 100 cm, then $\frac{78\text{cm}}{1}\times\frac{1\text{m}}{100\text{cm}}$

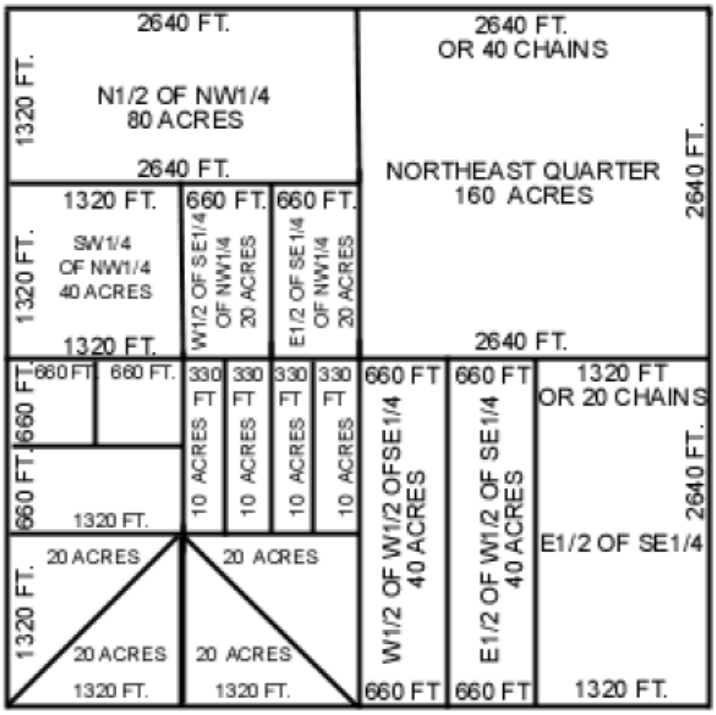
Step 6. $\frac{78\text{cm}}{1}\times\frac{1\text{m}}{100\text{cm}}=\frac{78}{100}$

Step 7. $\frac{78\text{cm}}{1}\times\frac{1\text{m}}{100\text{cm}}=\frac{78}{100}=0.78$

Step 8. $\frac{78\,\cancel{\text{cm}}}{1}\times\frac{1\text{m}}{100\,\cancel{\text{cm}}}=\frac{78}{100}=0.78\text{m}$

Notice how this algorithm automatically "takes care" of whether you multiply or divide (this occurs in Step 5). If you decide to use this algorithm, you still should ask yourself, "Which unit is larger?" If you ever get lost in making conversions, you can always resort to this guaranteed eight-step method.

Lengths in the Bible			
finger	Jeremiah 42:21	=	1 digit
handbreadth	Exodus 25:25	=	1 hand
span	Exodus 28:16	=	1 span
cubit	Genesis 6:15	=	2 spans
fathom	Acts 27:28	=	4 cubits
reed	Ezekiel 11:3-5	=	11 feet
line	Ezekiel 11:3	=	80 cubits
furlong	Luke 24:13	=	1 Greek furlong
mile	Matthew 5:41	=	8 Greek furlongs
Sabbath Day's Journey	Acts 1:12	=	1 or 2 miles



1 Section of Land = 640 acres (United States and Canada)

THE METRIC SYSTEM II

In our last chapter, we introduced the metric system and its decimal structure. There are three key prefixes that are commonly used: kilo-, centi-, and milli. The following table is a complete list of prefixes that scientists use:

Prefix	*Symbol*	*Meaning*
yotta-	Y	1,000,000,000,000,000,000,000,000 or 10^{24} times larger than the unit it precedes
zeta-	Z	1,000,000,000,000,000,000,000 or 10^{21} times larger than the unit it precedes
exa-	E	1,000,000,000,000,000,000 or 10^{18} times larger than the unit it precedes
peta-	P	1,000,000,000,000,000 or 10^{15} times larger than the unit it precedes
tera-	T	1,000,000,000,000 or 10^{12} times larger than the unit it precedes
giga-	G	1,000,000,000 or 10^{9} times larger than the unit it precedes
mega-	M	1,000,000 or 10^{6} times larger than the unit it precedes
kilo-	k	1000 or 10^{3} times larger than the unit it precedes
deci-	d	10 times smaller ($\frac{1}{10}$ or 10^{-1}) than the unit it precedes
centi-	c	100 times smaller ($\frac{1}{100}$ or 10^{-2})than the unit it precedes
milli-	m	1000 times smaller ($\frac{1}{1000}$ or 10^{-3})than the unit it precedes
micro-	μ or mc	1,000,000 times smaller ($\frac{1}{1,000,000}$ or 10^{-6}) than the unit it precedes
nano-	n	1,000,000 times smaller ($\frac{1}{1,000,000,000}$ or 10^{-9}) than the unit it precedes
pico-	p	1,000,000,000 times smaller ($\frac{1}{1,000,000,000,000}$ or 10^{-12}) than the unit it precedes
femto-	f	1,000,000,000,000 times smaller ($\frac{1}{1,000,000,000,000,000}$ or 10^{-15}) than the unit it precedes
atto-	a	1,000,000,000,000,000 times smaller ($\frac{1}{1,000,000,000,000,000,000}$ or 10^{-18}) than the unit it precedes
zepto-	z	1,000,000,000,000,000,000 times smaller ($\frac{1}{1,000,000,000,000,000,000,000}$ or 10^{-21}) than the unit it precedes
yocto-	y	1,000,000,000,000,000,000,000 times smaller ($\frac{1}{1,000,000,000,000,000,000,000,000}$ or 10^{-24}) than the unit it precedes

We also noted that measurement can be classified in six ways:

1. Extension.
2. Capacity.
3. Weight.
4. Time.
5. Angles.
6. Value (e.g., money).

METRIC SYSTEM II

In the last chapter, we discussed measurement of extension and capacity. We have already been introduced to measure of value (e.g., the decimal money system), time, and angles. Time and angle measurements are *not* metric. They reflect various bases: Babylonian (base 60) and Biblical (base 7, the creation days of Genesis 1, and the ordered pattern of the solar system) roots.

For angles, we have these relationships:

360° (degrees)	=	1 circle
60′ (minutes)	=	1°
60″ (seconds)	=	1′

For time, we have these relationships:

1000 years	=	1 millennium
100 years	=	1 century
10 years	=	1 decade
12 calendar months	=	1 common year
52 weeks	=	1 common year
365 days	=	1 common year
366 days	=	1 leap year[1]
7 days	=	1 week
24 hours	=	1 day
60′ (minutes)	=	1 hour
60″ (seconds)	=	1 minute

Interesting facts about 365:
$365 = 10^2 + 11^2 + 12^2$
$365 = 13^2 + 14^2$

One second is officially defined (SI standard) as 9,192,631,770 cycles (energy changes) in a cesium-133 atom! The history of the calendar and the relationship between the calendar and the Earth-Moon-Sun system is rich and fascinating.[2]

Sundial, Source: iStockPhoto

Stopwatch, Source: iStockPhoto

We introduced measures of capacity in the last lesson when we talked about liters and kitchen measures. Let's note these non-metric and historically related relationships (they are still used in the United States). First, here is a standard table of *liquid* measure.

4 gills[3] (gi)	=	1 pint (pt)
2 pints	=	1 quart[4] (qt)
4 quarts	=	1 gallon[5] (gal)
31.5 gallons	=	1 barrel

[1] Every year whose number of days is divisible by 4 is a leap year, except centennial years (e.g., 1800). However, centennial years divisible by 400 *are* leap years (1600 and 2000 were leap years). All other years are common years. An extra day is added to the month of February in leap years.

[2] For a good explanation of this history, see David Ewing Duncan, *Calendar: Humanity's Epic Struggle to Determine a True and Accurate Year* (New York: Avon Books, 1998).

[3] A *gill* in ancient Rome was a water or wine pot (equivalent to a drinking glass).

[4] A *quart* means "a fourth part" in Latin.

[5] A *gallon* means "a bucket" or "a jug" in Latin.

Apothecaries' *liquid* measure is used in prescribing and in compounding liquid *medicines*.[6]

60 minims[7] (drops or grains)	=	1 fluid dram[8]
8 fluid drams	=	1 fluid ounce (fl oz)
16 fluid ounces	=	1 pint
8 pints	=	1 gallon

Dry measure, as we have noted, is used in measuring non-liquid objects like grain, fruit, roots, etc. Some of our homework problems in the past have used these units.

2 pints	=	1 quart
8 quarts	=	1 peck[9]
2 gallons	=	1 peck
4 pecks	=	1 bushel[10]
1 bushel	=	2150.42 in^3

In review, metric measures of capacity are:

1000 microliters (μL)	=	1 milliliter (mL)	1 μL ≈ crystal of table salt
10 milliliters	=	1 centiliter (cL)	1 mL ≈ 20 drops of water
10 centiliters	=	1 deciliter (dL)	
10 deciliters	=	1 liter	1 L ≈ quart of milk
10 liters	=	1 decaliter (daL)	
10 decaliters	=	1 hectoliter (hL)	
10 hectoliters	=	1 kiloliter (kL)	

Finally, let's look at non-metric measures of weight. Going back into history, the ancient Babylonians used a cubic foot of rain water as their standard unit of weight, the **talent**.[11] Holding true to their base 60 system, the primary subdivision of a talent, a *maneh*, was $\frac{1}{60}$ of a talent. *Talent*, as a measure of weight, was also used by the ancient Hebrews (equal to about 94 pounds) and the Greeks (equal to about 57 pounds).

Weight, near the surface of the earth, is now defined the measure of gravity, and varies according to the quantity of mass an object contains. There are three types of non-metric measurement of weights:

(1) Troy weight.
(2) Apothecaries' weight
(3) **Avoirdupois** weight.[12]

[6] *Apothecary* is a Latin word meaning "storehouse or shop." In medieval times, an apothecarius, was a "seller of spices and drugs."
[7] *Minim* means "very small" in Latin.
[8] *Dram* is a derivative of *drachma* (meaning a "handful"), a standard silver coin in Ancient Greece. Note that the drachma was both a standard unit of both weight and value.
[9] *Peck* comes from *pick* and could mean a container used to hold fruit as it is picked or a box that can be easily carried.
[10] A *bushel*, from the Middle English, means a "box."
[11] David E. Smith, *History of Mathematics* (New York: Dover, [1925, 1953] 1958), 2:635.
[12] *Avoirdupois*, in Latin, literally means "to have earlier." In Middle English, this word was written as *aver de poiz* meaning "goods of weight." In measurement, it means a "property of weight." In the 16th century, it was commonly called "haberdepoise."

METRIC SYSTEM II

Troy[13] weight, historically, was used primarily by goldsmiths to weigh precious metals like gold, silver, or jewels.

24 grains[14]	=	1 pennyweight (pwt)[15]
20 pennyweights	=	1 ounce
12 ounces	=	1 pound (lb)

The fact that 12 ounces equal 1 troy pound comes from the ancient Roman standard of 1 libra (pound) = 12 unciae (parts or units). In ancient Rome, 1 ounce was equal to 412 grains.

A **carat** (from a Greek word meaning "little horn") was historically set to be equal to 12 troy grains. For the ancient Greeks, four *grains* of barley was equivalent to the carob seed or *keration* (a derivative of carat). *Carat* is also used to express the *fineness* of gold, and means "a twenty-fourth part." For example, when a piece of gold is said to be 18 carats fine, it means it has 18 parts pure gold and 6 parts of another metal. A piece of gold that is 24 carats fine is *pure* gold.

Apothecaries' *dry* measure is used in prescribing and in compounding dry medicines.

Silver coin = one troy ounce
Source: James Nickel

20 grains	=	1 scruple[16]
3 scruples	=	1 dram
8 drams	=	1 ounce
12 ounces	=	1 pound

Avoirdupois (abbreviated av) weight is used for general purposes, primarily for weighing heavier or coarse objects. It is also the basis for weighing letters and parcels to determine the cost of postage.

16 ounces (oz)	=	1 pound (1 kg ≈ 2.205 lb)
14 pounds	=	1 stone (used in British commonwealth countries)
100 pounds	=	1 hundred-weight (cwt)
20 hundred-weight (2000 pounds)	=	1 ton[17] or short ton (United States measurement)
2240 pounds	=	1 long ton (British measurement)

On the European continent, a troy pound was often called a "pound merchant," thus signifying its use in commercial transactions.

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[13] *Troy* comes from Troyes, a town in France, where it was standard of weight introduced due to the popularity of medieval fairs (ca. 13th century).

[14] *Grain*, in Latin, means "seed." In ancient England, one *grain* was equal to the weight of a barleycorn or a barley grain. In 17th century England, 24 barley grains, dry and taken out of the middle of the ear, were equal to the weight of one penny coin. A *troy grain* was equal to $\frac{1}{24}$ pennyweight.

[15] *Pennyweight* is a weight equal to that of a standard penny coin.

[16] *Scruple* comes from an ancient Roman unit of weight (equivalent to the weight of a small pebble).

[17] *Ton* comes from the Latin *tunna* meaning cask.

In contrast to these three types of weights, metric measures of weight are based upon the gram. 1 gram (g) = 15.433 troy grains = 0.03527 av oz.

1000 micrograms (µg or mcg)[18]	=	1 milligram (mg)	
10 milligrams	=	1 centigram (cg)	
10 centigrams	=	1 decigram (dg)	
10 decigrams	=	1 gram	1 g ≈ 1 paper clip; 28 g = 1 oz
10 grams	=	1 decagram (dag)	
10 decagrams	=	1 hectogram (hg)	
10 hectograms	=	1 kilogram (kg)	
1000 kilograms	=	1 metric ton or tonne	1 tonne = 1.1023 short tons

Food provides nutrients for the human body. The amount of nutrients that a body needs is usually measured in grams or a fraction of a gram. The **International Unit** (abbreviated IU) is a unit of measurement for the amount of a substance, based on measured biological activity or effect. The precise definition of one IU differs from substance to substance and is established by international agreement for each substance. There is no equivalence among different substances; for example, one IU of vitamin E does not contain the same number of milligrams as one IU of vitamin A.

Recommended Daily (ca. 2005) allowances of vitamins and minerals	
Nutrient	*Amount*
Vitamin A	5000 IU
Vitamin C	60 mg
Thiamin	1.5 mg
Riboflavin	1.7 mg
Niacin	20 mg
Calcium	1 g
Iron	19 mg
Vitamin D	400 IU
Vitamin E	30 IU
Vitamin B6	2 mg
Folic acid	0.4 mg
Vitamin B12	6 mcg
Phosphorus	1 g
Iodine	150 mcg
Magnesium	400 mcg
Zinc	15 mcg
Copper	2 mg
Biotin	0.3 mg
Panothenic acid	10 mg

Before we complete this measurement survey, let's note a few more relationships

Railroad tracks:

[18] The abbreviation µg is often used in scientific literature, but hospitals generally do not use this abbreviation in handwritten orders due to the risk that the Greek letter µ could be mistaken for an *m*, resulting in a *thousandfold* overdose. The abbreviation *mcg* is recommended instead.

METRIC SYSTEM II

Standard gauge: 4 feet 8½ inches, the distance between two rails of a railway track. Over half of the world's railway tracks are built to this gauge.

Broad gauge: 7 feet ¼ inch. High-speed trains in Europe use this gauge.

Size of type print:

1 point (type size) ≈ 0.138 in ≈ 0.353 mm (72 points = 1 inch, 28 points = 1 cm).

1 pica (type size) = 12 points ≈ 4 mm.

Computer display screens:

1 pixel, a contraction of "picture element," is a unit of display in the memory of a computer (or other digital system) or on a video screen. The earliest computer screens could only show a pixel as "on" or "off." These were followed by systems with higher resolutions (320 × 200 pixels in the 1980s) with a multiple of 8 or 16 colors. Modern systems usually have a screen resolution of 1024 × 768 or higher, and the ability to display millions of different colors. Very inexpensive digital cameras can capture images with *millions* of pixels.

1 voxel is the three-dimensional equivalent of a pixel and widely used to represent scientific data (e.g., medical imaging such as M.R.I. scans[19]) and, less often, for certain types of computer games.

British Monetary Measures (before 15 February 1971):[20]

2 Farthings = 1 Halfpenny (ha'penny).

2 Halfpence = 1 Penny.

2 Pence (d) = Tuppence.

6 Pence = Sixpence, often referred to as a tanner.

12 Pence (written as 12d) = 1 Shilling (s), often referred to as bob (e.g., six bob). 1 shilling was written as 1 s or 1/-

2 shillings = 1 Florin, or two bob bit.

2 Shillings and 6 Pence = 1 Half Crown.

5 shillings = 1 Crown.

20 Shillings = 1 Pound Sterling (£), often referred to as a quid or guinea.[21]

3 d
Source: James Nickel

2 pounds 14 shillings and five pence was been written as £2 14s 5d or 2/14/5.

1 s or 1/-
Source: James Nickel

British Common Law Measure:

1 jury = 12 people.

Counting certain types of objects:

1 dozen[22] = 12 objects.

1 gross[23] = 12 dozen.

1 great gross = 12 gross.

1 score[24] = 20 objects.

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[19] M.R.I. is an abbreviation of three words: "magnetic resonance imaging."

[20] This monetary system is an interesting combination of the vigesimal (base 20) and duodecimal (base 12) methods of counting. The English Pounds, Shillings, Pence symbolized by £sd, derived from LSD (in Latin, "Librae, Solidi, Denarii").

[21] Named after African Guinea where, in the 17th century, gold was mined for English coins.

[22] *Dozen* is the Middle English derivation of the Latin *duodecim,* meaning "twelve."

[23] *Gross* comes from the Middle English derivation of the Latin *grossus,* meaning "large, thick, coarse."

[24] *Score*, from the Middle English, means "group of twenty." Four score years would be 80 years. In Scandinavian countries, hens lay scores of eggs and are hence "vigesimal animals." In contrast, "English-speaking hens" are duodecimal and they lay their eggs "by the dozen."

> Four score and seven years ago our fathers brought forth on this continent a new nation ...
>
> Abraham Lincoln, Gettysburg Address (1863)

Paper trade:
24 sheets = 1 quire.[25]
20 quires = 1 ream.[26]
2 reams = 1 bundle.
5 bundles = 1 bale.

International Paper size:

Designation	Size in mm	Size in inches	Designation	Size in mm	Size in inches
A0	841by 1198	33.1 by 46.8	B0	1000 by 1414	39.4 by 55.7
A1	584 by 841	23.4 by 33.1	B1	707 by 1000	27.8 by 39.4
A2	420 by 594	16.5 by 23.4	B2	500 by 707	19.7 by 27.8
A3	297 by 420	11.7 by 16.5	B3	353 by 500	13.9 by 19.7
A4	210 by 297	8.3 by 11.7	B4	250 by 353	9.8 by 13.9
A5	148 by 210	5.8 by 8.3	B5	176 by 250	6.9 by 9.8
A6	105 by 148	4.1 by 5.8	B6	125 by 176	4.9 by 6.9
A7	74 by 105	2.9 by 4.1	B7	88 by 125	3.5 by 4.9
A8	52 by 74	2.1 by 2.9	B8	62 by 88	2.4 by 3.5
A9	37 by 52	1.5 by 2.1	B9	44 by 62	1.7 by 2.4
A10	26 by 37	1 by 1.5	B10	31 by 44	1.2 by 1.7

Bookbinding:
72 words = 1 folio[27] (or a sheet of legal paper, record, or document).
2 leaves = 1 folio (4 pages).
2 folios = 1 quarto[28] (8 pages).
2 quartos = 1 octavo[29] (16 pages).
2 octavos = 1 sixteenmo (32 pages).
2 sixteenmos = 1 thirty-twomo (64 pages).

There are many more units used in scientific measurements. For example, there are energy units, force units, pressure units, atomic units, and astronomical units. Some of these units can be very large (astronomical) and some can be very small (atomic).

There are seven foundational base units in the SI system. We have only discussed three. The other four are measurements common to studies in physics and chemistry.

[25] *Quire* comes from the Latin *quarternum,* meaning "set of four sheets."
[26] *Ream*, from Middle English, means "bale." A ream of paper now consists of 500 sheets (instead of 480).
[27] *Folio*, in Latin, means "leaf or sheet."
[28] *Quarto* is the singular form of the Latin *quartus* and means "fourth."
[29] *Octavo* is Latin for "eighth."

METRIC SYSTEM II

Quantity measured	*Unit*	*SI symbol*
Length	meter[30]	m
Mass	kilogram[31]	kg
Time	second	s
Electric current	ampere[32]	A
Thermodynamic temperature	kelvin[33]	K
Amount of substance	mole[34]	mol
Luminous intensity	candela[35]	cd

Dry Measures in the Bible			
1 pot	Mark 7:4	=	1.5 pints
1 cab	II Kings 6:25	=	2 pints
1 chenix	Revelation 6:6	=	2 pints
1 omer	Exodus 16:36	≈	3 pints
1 tenth-deal	Exodus 29:4	≈	3 pints
1 seah	Matthew 13:33	≈	2 gallons
1 bath or ephah	Ezekiel 45:11	≈	6 gallons
1 homer	Numbers 11:32; Hosea 3:2	≈	16 gallons

Liquid Measures in the Bible			
1 log	Leviticus 16:10	=	0.75 pint or six filled egg-shells
1 hin	Exodus 29:4	=	10 pints
1 bath	I Kings 7:26	=	60 pints or 6 hins
1 firkin	John 2:6	=	9 gallons
1 cor or homer	Ezekiel 45:14; Isaiah 5:10	=	10 baths

Money in the Bible				
quadrans	copper	Mark 12:42	=	2 mites (leptons) or ¼ of an as
assyrium (penny)	copper	Matthew 10:27	=	4 quadrantes or 1 as
denarius (drachma)	silver	Matthew 20:2	=	16 asses (day's wage for a laborer)
bekah (didrachma)	silver	Exodus 38:26	=	2 drachma
shekel (tetradrachma or silverling)	silver	Genesis 23:16; Matthew 17:27	=	2 bekah or 2 slater (11 grams)
mineh	silver	Luke 19:13	=	50 shekels (0.6 kilogram) or 30 shek-

[30] One *meter* was originally set to be 1/10-millionth the distance from the North Pole to the equator through Paris. Since 1983 one meter has been defined as the distance traveled by light in a vacuum in 1/299,792,458 second.
[31] One *kilogram* is equivalent to a cylinder of platinum-iridium alloy held by the International Bureau of Weights and Measures at Sevres, near Paris, France. This is the only remaining artifact-based standard of measure still in use.
[32] One *ampere* is the magnitude of a current that results in a force equal to 0.0000002 newton (N) per meter of length. A *newton* is a unit of force and is equal to the force that produces an acceleration of one meter per second per second on a mass of one kilogram. We are into the realm of physics here!
[33] One *Kelvin* is the point immediately above absolute zero, where all atomic activity ceases (the fraction 1/273.16 of the thermodynamic temperature of the triple point of water). K = C + 273.16° (where C = degrees Celsius or degrees centigrade).
[34] One *mole* is the amount of a substance that contains as many elementary entities as there are atoms in 12 grams of carbon-12.
[35] One *candela* is the luminous intensity of a source that emits monochromatic radiation of frequency 540,000,000,000,000 hz (hertz) or vibrations per second.

Money in the Bible				
				els (the mina) in the New Testament
talent	silver	II Kings 5:22	=	60 minehs (36 kilograms)

GOING BACK AND FORTH

In our definition of metric measures, I introduced several connections to the British Imperial System. In review, here are a few:

1 cm	≈	0.3937 in.
1 m	≈	3.281 ft.
1 km	≈	0.62137 miles
1 hectare	≈	2.471 acres
1 L	≈	0.908 dry quarts
1 L	≈	1.0567 quarts (liquid)
1 cup	=	240 mL
1 kg	≈	2.205 pounds (av)
1 tonne	=	1.1023 short tons

These **quantitative**[1] connections between the Metric System and the British Imperial System are called **conversion factors**. They immediately inform us of comparisons. For example, which unit is larger, an inch or a centimeter? Since 1 cm is a fraction of an inch, then 1 cm < 1 inch. Since 1 meter is a little more than 3 feet, then 1 m > 1 ft.

Which is larger, 1 km or 1 mile? Since 1 km is a fraction of a mile, then 1 km < 1 m.

Which is larger, 1 hectare or 1 acre? Since 1 hectare is about 2.5 acres, then 1 hectare > 1 acre.

Which is larger, 1 liter or 1 quart? Since 1 liter is a fraction of a quart, then 1 L < 1 quart.

Which is larger, 1 cup or 1 milliliter? Since 1 cup is 240 mL, then 1 cup > 1 mL (not even close!).

Which is larger, 1 lb or 1 kg? Since 1 kg is about 2.2 lb, then 1 kg > 1 lb.

Which is larger, 1 tonne or 1 short ton? Since 1 tonne is about 1.1 times larger than a short ton, then 1 tonne > 1 short ton.

Do you see a pattern? If unit A is equal to a fraction (less than 1) of unit B, then unit A < unit B, otherwise unit A > unit B (we are, of course, assuming that unit A ≠ unit B).

We can apply this pattern to conversions within the Metric System itself. For example, since 1 km = 1000 m, then 1 km > 1 m. Since 1 mm = 0.001 m, then 1 mm < 1 m.

Let's try some conversions between the

Standard Atmosphere: the pressure equivalent to that exerted by a 760 mm column of mercury at 0°C (32°F), at sea level, and at standard gravity (32.174 ft/sec²), the force of attraction by which terrestrial bodies tend to fall toward the center of the earth. Atmospheric pressure is the weight of a column of air per area unit as measured from the top of the atmosphere to the reference point being measured. Atmospheric pressure *decreases* as altitude *increases*.
Equivalents to 1 atmosphere are:
76 cm (760 mm) of mercury
29.921 inches of mercury
10.3322 meters of water
406.782 inches of water
33.899 feet of water
14.696 pounds of force per square inch
2116.2 pounds of force per square foot
1.033 kilograms of force per square centimeter

[1] *Quantitative* means "of or pertaining to the describing or measuring of quantity."

Metric System and the British Imperial System. Recall our steps that we used to make conversions between units within the Metric System, for these steps are applicable with any type of conversion.

Step 1. Ask yourself, "Which unit is the larger unit?"

Step 2. If you are converting from a *larger unit to a smaller unit*, then *multiply* by the conversion relationship (also called the "unit multiplier"). Why? You need *more* of the smaller unit to make up the larger unit. Another way to say this is if you are converting *down* the scale to a *smaller* unit of measure, the quantity (or the number of units) must get *larger*.

Step 3. If you are converting from a *smaller unit to a larger unit*, then *divide* by the unit multiplier. Another way to say this is if you are converting *up* the scale to a *larger* unit of measure, the quantity (or number) must get *smaller*.

Or, we can use this eight-step approach:

Step 1. If the unit of measurement that you are converting is a whole number, put it over 1. If it is a fraction, first convert it to a decimal and then put it over 1.

Step 2. Place a multiplication sign next to the right.

Step 3. Draw a fraction bar to the right.

Step 4. Put in the units of measurement. The unit of measurement to be removed is written in the denominator. The unit of measurement to be converted is written in the numerator.

Step 5. Enter the unit multiplier (1 km = 1000 m) matching the units appropriately.

Step 6. Multiply disregarding the units.

Step 7. Reduce or simplify the fraction if necessary.

Step 8. Cancel like units of measurement and carry over the remaining unit of measurement to the answer.

Let's ask a question: How many feet are there in 3.6 km? We know that since 1 km = 1000 m (1 km > 1 m), then, since 1000 is our unit multiplier, 3.6 km = $\frac{3.6\,\cancel{\text{km}}}{1}\times\frac{1000\text{m}}{\cancel{\text{km}}}=3600\text{m}$ (we multiply since we are converting from a larger unit to a smaller unit). We also know that 1 m ≈ 3.3 feet. Hence, 3.281 is our unit multiplier. Which unit is larger, meters or feet? Since 1 m > 1 ft, then $\frac{3600\,\cancel{\text{m}}}{1}\times\frac{3.281\,\text{ft}}{\cancel{\text{m}}}\approx 11{,}812\,\text{ft}$ (we multiply since we are converting from a larger unit to a smaller unit). Notice especially how the units cancel.

Now let's see if we can convert 10 lb, 4 oz. (avoirdupois) into kg. As in our previous example, we will have to perform several steps to get our answer. Let's go ahead and convert 10 lb, 4 oz. to pounds in decimal format. This means we have to convert 4 oz. to pounds. Since, in troy weight, 1 lb = 16 oz., then, 1 lb > 1 oz. Converting from ounces to pounds means we are converting from a smaller unit to a larger unit. Hence, we must divide:

$\frac{4\,\cancel{\text{oz}}}{1}\times\frac{\text{lb}}{16\,\cancel{\text{oz}}}=\frac{\overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}}}\text{lb}=\frac{1}{4}\text{lb}=0.25\,\text{lb}$. Hence, 10 lb, 4 oz = 10.25 lb. We now want to convert from pounds to kilograms. Our unit multiplier is 2.2 (1 kg ≈ 2.205 lb). We know that 1 kg > 1 lb and this means that we are converting again from a smaller unit to a larger unit. Hence, we must divide:

$\frac{10.25\,\cancel{\text{lb}}}{1}\times\frac{\text{kg}}{2.205\,\cancel{\text{lb}}}\approx\frac{10.25}{2.205}\text{kg}\approx 4.649\text{kg}$. Notice again how the units cancel.

Here is an additional table of unit conversion factors.

1 yd	≈	0.9144 m
1 in^2	≈	6.452 cm^2

GOING BACK AND FORTH

1 m^2	≈	2.590 km^2
1 in^3	≈	16.387 cm^3
1 quart (dry)	≈	1.101 L
1 oz (av)	≈	28.35 g
1 oz (troy)	≈	31.1035 g
1 lb (troy)	≈	0.3732 kg

Finally, here is a table that summarizes exact and approximate (fractional) unit multipliers for popular units. The "exact" decimal representation is *rounded* and correct to the number of decimals shown.

Unit	*Equivalent*		*Unit*	*Equivalent*	
	Exact	*Approximate*		*Exact*	*Approximate*
1 acre	0.4047 hectare	$\frac{2}{5}$	1 mm	0.03937 in	$\frac{1}{25}$
1 bushel	35.24 L	$35\frac{1}{5}$	1 oz (av)	28.35 g	$28\frac{1}{3}$
1 cm	0.3937 in	$\frac{2}{5}$	1 oz (troy)	31.10 g	31
1 cm^3	0.0610 in^3	$\frac{1}{17}$	1 peck	8.809 L	$8\frac{4}{5}$
1 ft^3	0.0283 m^3	$\frac{1}{36}$	1 pint (liquid)	0.4732 L	$\frac{1}{2}$
1 in^3	16.387 cm^3	$16\frac{2}{5}$	1 pound (av)	0.4536 kg	$\frac{4}{9}$
1 m^3	1.308 yd^3	$1\frac{1}{3}$	1 quart (dry)	1.101 L	$1\frac{1}{10}$
1 m^3	35.31 ft^3	$35\frac{1}{3}$	1 quart (liquid)	0.9464 L	1
1 yd^3	0.7645 m^3	$\frac{3}{4}$	1 cm^2	0.1550 in^2	$\frac{1}{6}$
1 foot	30.48 cm	$30\frac{1}{2}$	1 ft^2	0.0929 m^2	$\frac{1}{11}$
1 gallon	3.785 L	$3\frac{4}{5}$	1 in^2	6.452 cm^2	$6\frac{1}{2}$
1 grain (av)	0.648 g	$\frac{1}{15}$	1 mi^2	259 hectares	260
1 g	15.43 grains (av)	$15\frac{1}{2}$	1 m^2	1.196 yd^2	$1\frac{1}{5}$
1 hectare	2.471 acres	$2\frac{1}{2}$	1 m^2	10.76 ft^2	$10\frac{3}{4}$
1 in	2.54 cm	$2\frac{1}{2}$	1 ft^2	0.093 m^2	$\frac{9}{100}$

Unit	*Equivalent*		*Unit*	*Equivalent*	
1 kg	2.205 lb	$2\frac{1}{5}$	1 yd^2	0.8361 m^2	$\frac{4}{5}$
1 km	0.6215 mi	$\frac{5}{8}$	1 ton	0.9072 tonnes	$\frac{9}{10}$
1 L	0.9081 quart (dry)	$\frac{9}{10}$	1 long ton	1.017 tonnes	1
1 L	1.057 quarts (liquid)	1	1 tonne	1.102 tons	$1\frac{1}{10}$
1 m	3.281 ft	$3\frac{1}{4}$	1 tonne	0.9842 long ton	1
1 mi	1.6093 km	$1\frac{3}{5}$	1 yd	0.9144 m	$\frac{9}{10}$

Board feet: Lumber is sold by a "feet board measure" or "board foot" rating. 1 board foot = 144 cubic inches (in^3). For example, 1 board foot = 12 inch by 12 inch by 1 inch or 2 inch by 6 inch by 12 inch. Board feet is determined by either of these two formulas:

1. [thickness (in) × face width (in) × length (in)]/144
2. [thickness (in) × face width (in) × length (ft)]/12

Can you see why?

The following are quick approximations for calculating board feet:

Board (ft × ft)	*Calculation*
1 × 4	Divide linear length (ft) by 3
1 × 6	Divide linear length (ft) by 2
1 × 8	Multiply linear length (ft) by $\frac{2}{3}$
1 × 12	Linear length (ft) = board feet
2 × 4	Multiply linear length (ft) by $\frac{2}{3}$
2 × 6	Linear length (ft) = board feet
2 × 8	Multiply linear length (ft) by $\frac{4}{3}$
2 × 12	Multiply linear length (ft) by 2

Can you see why?